

The South African Mathematical Olympiad
Third Round 2006
Senior Division (Grades 10 to 12)
Time : 4 hours

1. Reduce the fraction

$$\frac{2121212121210}{1121212121211}$$

to its simplest form.

2. Triangle ABC has $BC = 1$ and $AC = 2$. What is the maximum possible value of \widehat{A} ?
3. Determine all positive integers whose squares end in 196.
4. In triangle ABC, $AB = AC$ and $\widehat{AC} = 100^\circ$. D is on AC such that $\widehat{ABD} = \widehat{CBD}$. Prove that $AD + DB = BC$.
5. Find the number of subsets X of $\{1, 2, \dots, 10\}$ such that X contains at least two elements and such that no two elements of X differ by 1.
6. Consider the function f defined by

$$f(n) = \frac{1}{n} \left(\left\lfloor \frac{n}{1} \right\rfloor + \left\lfloor \frac{n}{2} \right\rfloor + \dots + \left\lfloor \frac{n}{n} \right\rfloor \right)$$

for all positive integers n. ($\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

Prove that:

- (a) $f(n+1) > f(n)$ for infinitely many n
- (b) $f(n+1) < f(n)$ for infinitely many n