SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2011 SECOND ROUND
SENIOR SECTION: GRADES 10, 11 AND 12

17 May 2011 **Time: 120 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.*
   2.2. *For each incorrect answer one mark is deducted. There is no penalty for unanswered questions.*
3. *You must use an HB pencil. Rough work paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Figures are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so.
Draai die boekie om vir die Afrikaanse vraestel.

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. As a decimal number 6.28% is equal to

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. The value of $1 + \dfrac{1}{3 + \dfrac{1}{2}}$ is

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{6}{7}$ (E) $\frac{9}{7}$

3. The tens digit of the product $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

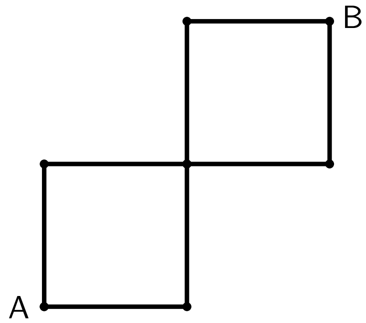
PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

Part A: Four marks each

1. If you start at 1 and count in 3's you obtain the sequence 1, 4, 7, 10, 13, $\cdots$. The 100th number in this sequence is

(A) 298 (B) 301 (C) 304 (D) 307 (E) 310

2. By moving only upwards or to the right, the number of different ways to get from A to B is



(A) 4 (B) 6 (C) 8 (D) 10 (E) 12

3. Fifteen coins are separated into four piles so that each pile has a different number of coins. The least possible number of coins in the largest pile is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

4. If M and N are two different integers selected from the integers 1 to 50, then the greatest possible value of $\dfrac{M+N}{M-N}$ is

(A) 51 (B) 99 (C) 109 (D) 125 (E) 144

5. To make orange paint, 3 parts of red paint are mixed with 2 parts of yellow paint. To make green paint, 2 parts of blue paint are mixed with 1 part of yellow paint. If equal volumes of green and orange are mixed, then the proportion of yellow paint in the mixture is

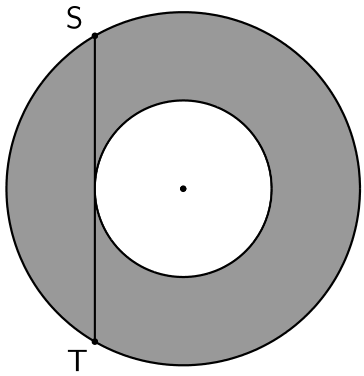
(A) $\dfrac{3}{16}$ (B) $\dfrac{1}{4}$ (C) $\dfrac{11}{30}$ (D) $\dfrac{3}{10}$ (E) $\dfrac{7}{12}$

Part B: Five marks each

6. A restaurant can accommodate at most 400 people, which includes the guests and the waiters. Each waiter can serve a maximum of 12 guests. The greatest number of guests that can be served is

(A) 358 (B) 360 (C) 369 (D) 372 (E) 375

7. In the diagram, ST is a tangent to the smaller of the two concentric circles. If ST = 40 cm, then the area, in cm^2, of the shaded region between the two circles is



(A) $144\,\pi$ (B) $196\,\pi$ (C) $225\,\pi$ (D) $324\,\pi$ (E) $400\,\pi$

8. The product of two positive integers is 10 000. If neither of these two numbers has a zero as one of its digits, then their sum is

(A) 641 (B) 625 (C) 525 (D) 657 (E) 689

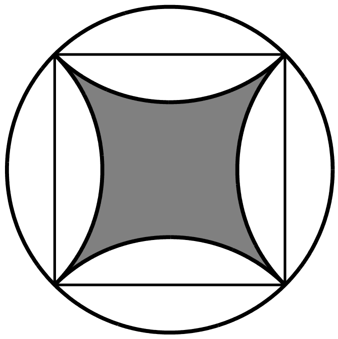
9. The first few Fibonacci numbers are **1**, **1**, **2**, **3**, **5**, **8**, 13, 21, ... Two identical dice have the first six of these numbers on their sides. If these two dice are rolled, then the probability that the sum of the two numbers thrown is also a Fibonacci number is

(A) $\frac{1}{3}$ (B) $\frac{7}{18}$ (C) $\frac{4}{9}$ (D) $\frac{5}{18}$ (E) $\frac{1}{2}$

10. The power of 10 closest to the number $17 \times 18 \times 19 \times 20 \times 21 \times 22 \times 23$ is

(A) 10^6 (B) 10^7 (C) 10^8 (D) 10^9 (E) 10^{10}

11. A square is drawn inside a circle of radius 1. If the four flaps of the circle are folded over as shown, then the area of the shaded region is



(A) $\frac{1}{3}\pi$ (B) $\frac{1}{4}\pi$ (C) $\frac{1}{2}\pi - \frac{1}{\sqrt{2}}$ (D) $4 - \pi$ (E) $2 - \frac{1}{3}\pi$

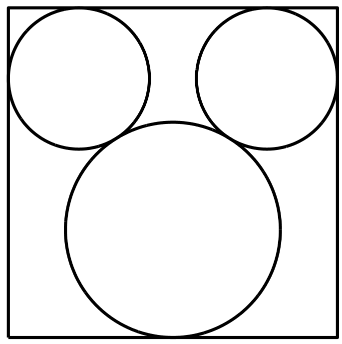
12. If two positive integers M and N satisfy $M^2 - N^2 = 2011$, then the value of N is

(A) 99 (B) 1005 (C) 1011 (D) 1200 (E) 1584

13. The notation $n!$ denotes the product of the integers from 1 to n. The greatest number of times that 100! can be divided by 3 without remainder is

(A) 33 (B) 39 (C) 42 (D) 45 (E) 48

14. Walt designs a cartoon character consisting of two small circles touching a large circle. The three circles fit inside a square as shown. If the radius of each of the small circles is 3 and the side length of the square is 14, then the radius of the large circle is



(A) $\frac{19}{4}$ (B) $\frac{13}{3}$ (C) $\frac{24}{5}$ (D) $\frac{9}{2}$ (E) $\frac{32}{7}$

15. Let x be any real number. The greatest possible value of $3^x - 9^x$ is

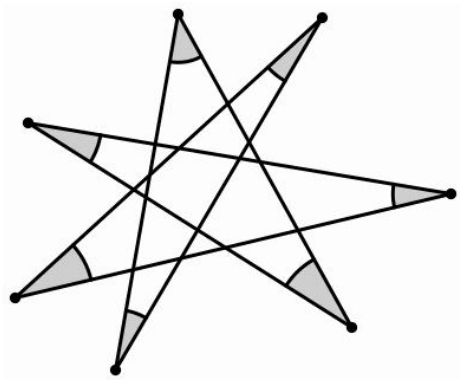
(A) $\frac{1}{4}$ (B) $3 - \sqrt{3}$ (C) $1 - \frac{1}{\sqrt{3}}$ (D) $\frac{1}{2}$ (E) $\sqrt{3} - 1$

Part C: Six marks each

16. If the five numbers $\left\{ \sqrt{11}-\sqrt{10}\ ;\ \sqrt{10}-3\ ;\ \frac{1}{6}\ ;\ \frac{\sqrt{10}}{20}\ ;\ \frac{\sqrt{11}}{22} \right\}$ are arranged from the smallest to the largest, then the one in the middle is

(A) $\sqrt{11}-\sqrt{10}$ (B) $\sqrt{10}-3$ (C) $\frac{1}{6}$ (D) $\frac{\sqrt{10}}{20}$ (E) $\frac{\sqrt{11}}{22}$

17. In the diagram, the sum of the angles at the vertices of the star is



(A) 90° (B) 135° (C) 150° (D) 180° (E) 360°

18. James was swimming in a river, against the current, when he lost his goggles. He continued swimming for 10 minutes, then decided to turn around and retrieve them. He caught up with his goggles, which floated downstream at the same constant speed as the river, at a distance 500 metres from where he lost them. If James swam at the same strength throughout, then the river was flowing at a speed of

(A) 0.5 km/h (B) 1 km/h (C) 1.5 km/h (D) 2 km/h (E) 3 km/h

19. In chess tournaments, players get 1 point for a win, $\frac{1}{2}$ point for a draw and 0 for a loss. In a recent round-robin tournament each pair of players met exactly once, and the top four scores were $4\frac{1}{2}$, $3\frac{1}{2}$, 3 and $1\frac{1}{2}$. The lowest score at that tournament was

(A) $1\frac{1}{2}$ (B) 1 (C) $\frac{1}{2}$ (D) 0 (E) impossible to determine

20. Four couples, including my partner and me, met for lunch. Several handshakes took place. No one shook hands with himself (or herself) or his (or her) partner, and no one shook hands with the same person more than once. After all the handshaking was completed I asked each person, including my partner, how many hands he or she had shaken. Each of these seven persons gave a different answer. How many hands did I shake?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 6