THE SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

SECOND ROUND 2008
SENIOR SECTION: GRADES 10, 11 AND 12

22 MAY 2008
TIME: 120 MINUTES
NUMBER OF QUESTIONS: 20

Instructions

1) Do not open this booklet until told to do so by the invigilator.
2) This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3) Scoring rules:
   a) Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.
   b) For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4) You must use an HB pencil. Rough paper, a ruler and an eraser are permitted. **Calculators and geometry instruments are not permitted.**
5) Diagrams are not necessarily drawn to scale.
6) Indicate your answers on the sheet provided.
7) Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.
8) Answers and solutions will be available at www.samf.ac.za in June.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO
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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0.5129, 0.9, 0.89, and 0.289 is

(A) 1.189
(B) 0.8019
(C) 1.428
(D) 1.179
(E) 1.4129

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Part A: Four marks each

1. The tens digit of the product $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

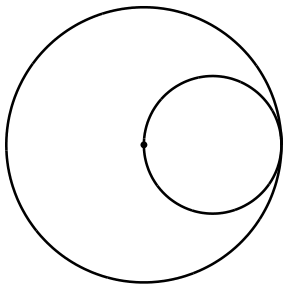
(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

2. The numbers from 1 to 5 are to be written in a 5×5 grid so that each number appears exactly once in each row and in each column. Some of the numbers have already been entered. What number can go in the square marked x?

			5	
1				2
			4	
2			x	

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

3. In the figure the diameter of the smaller circle is the radius of the bigger circle. The ratio of the area of the bigger circle to the area of the smaller circle equals



(A) π (B) 3 (C) 4 (D) 6 (E) 2π

4. $\sqrt{36^{36}}$ equals

(A) 6^2 (B) 6^6 (C) 6^{12} (D) 6^{18} (E) 6^{36}

5. What is the remainder when 200000000000000008 is divided by 3?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Part B: Five marks each

6. If 173 digits were used to number the pages of a book, starting at page 1, then the number of pages in the book is

(A) 89 (B) 90 (C) 91 (D) 92 (E) 94

7. If a and b are nonzero numbers such that a and b are the two roots of $x^2+ax+b=0$, then b equals

(A) -2 (B) -1 (C) 1 (D) 2 (E) 3

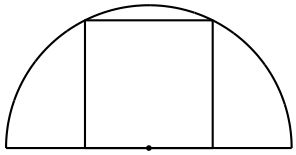
8. For how many integers n does $\sqrt{n}$ differ from 11 by less than 1?

(A) 1 (B) 2 (C) 16 (D) 43 (E) 56

9. What is the average distance between two different corners of a square of side 1?

(A) 1 (B) $\sqrt{2}$ (C) $1+\sqrt{2}$ (D) $\dfrac{\sqrt{2}+1}{3}$ (E) $\dfrac{\sqrt{2}+2}{3}$

10. A square is inscribed in a semicircle of radius 1 as shown. The area of the square is



(A) $\dfrac{3}{4}$ (B) $\dfrac{4}{5}$ (C) $\dfrac{5}{6}$ (D) $\dfrac{6}{7}$ (E) 1

11. Consider a square with area S and side length s, and an equilateral triangle with area D and side length d. If $\dfrac{D}{S}=\sqrt{3}$, then $\dfrac{d}{s}$ equals

(A) 3 (B) 1 (C) $\sqrt{2}$ (D) 2 (E) $\sqrt{3}$

12. Five straight lines are drawn. What is the maximum number of points of intersection?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

13. A regular polygon with 2008 sides and perimeter 1 has area approximately equal to

(A) $\frac{1}{20}$ (B) $\frac{1}{16}$ (C) $\frac{1}{12}$ (D) $\frac{1}{9}$ (E) $\frac{1}{8}$

14. The number of three-digit numbers that are divisible by 9 and contain no even digits is

(A) 10 (B) 11 (C) 12 (D) 13 (E) 14

15. The function $f(x)$ satisfies the equation

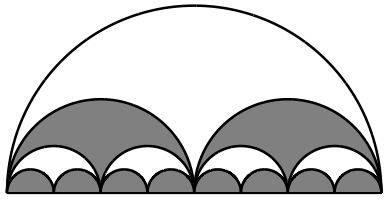
$$f(2^x) + xf(2^{-x}) = 1$$

for all values of x. The value of $f(2)$ is

(A) 0 (B) 1 (C) -1 (D) 2 (E) -2

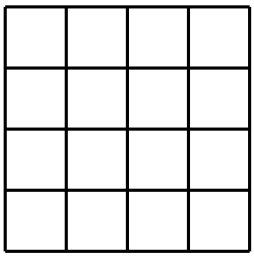
Part C: Six marks each

16. The diagram shows a white semicircle of radius r, inside of which two grey semicircles of radius $r/2$ are inscribed, inside of which four white semicircles of radius $r/4$ are inscribed, etc. If this pattern is continued indefinitely, what fraction of the original semicircle will eventually be white?



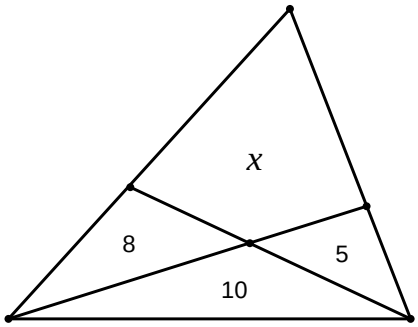
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{3}{5}$ (E) $\frac{4}{7}$

17. How many rectangles (of all sizes) does the 4×4 grid shown in the figure contain? (For example, a 2×2 grid contains nine rectangles.)



(A) 256 (B) 144 (C) 64 (D) 100 (E) 128

18. If a triangle is divided into four pieces with areas as shown, then the area x equals:



(A) 12.5 (B) 13 (C) 15 (D) 18 (E) 22

19. How many real solutions does the following equation have?

$$(x+1)(2^x - 1) = 1$$

(A) 0 (B) 1 (C) 2 (D) 3 (E) more than three

20. If $3 \times 2^a + 5^b + 7^c + 11^d = 2008$ with a, b, c, and d all non-negative integers, then $a + b + c + d$ equals

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10