THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION
Sponsored by HARMONY GOLD MINING

SECOND ROUND 2007

SENIOR SECTION: GRADES 10, 11 AND 12

15 MAY 2007

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 120 minutes to complete the question paper.
8. Answers and solutions will be available in June at: http://www.samf.ac.za/

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0.5129; 0.9; 0.89; and 0.289 is

(A) 1.189
(B) 0.8019
(C) 1.428
(D) 1.179
(E) 1.4129

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Part A: Four marks each

1. Eight thousand people die of AIDS worldwide every day. The number that die every two minutes is approximately

(A) 5 (B) 15 (C) 3 (D) 11 (E) 8

2. If a new mathematical operation $\bigtriangledown$ is defined by $a \bigtriangledown b = \dfrac{a}{a+b}$, then $1 \bigtriangledown (2 \bigtriangledown 3)$ equals

(A) $\dfrac{1}{2}$ (B) $\dfrac{2}{3}$ (C) $\dfrac{5}{7}$ (D) $\dfrac{3}{8}$ (E) $\dfrac{7}{9}$

3. A rectangular piece of paper has the property that if it is folded along the shorter midline, the resulting rectangle is similar to the original rectangle. The ratio of its length to its width is

(A) $\sqrt{2}$ (B)2 (C) $\sqrt{3}$ (D) 3 (E) 4

4. A bicycle wheel has a diameter of 80cm. If Ellané rides her bicycle for 120 km, then the number of rotations of her bicycle wheel is about

(A) 10 000 (B) 50 000 (C) 100 000 (D) 5 000 (E) 120 000

5. For how many integers b from 1 to 100 is there a positive integer a such that $a^3 = b^2$?

(A) 1 (B) 2 (C) 5 (D) 4 (E) more than 5

Part B: Five marks each

6. Two lines AB and CD are parallel and a distance 8 apart. Suppose that AD intersects BC in a point P between the lines. If $AB = 4$ and $CD = 12$, then the distance of P from the line CD is

(A) 3 (B) 2 (C) 4 (D) 6 (E) impossible to determine

7. If $3^x = 2$ which one of the following statements is false?

(A) $x < \dfrac{3}{4}$ (B) $x > \dfrac{4}{7}$ (C) $x < \dfrac{2}{3}$ (D) $x < \dfrac{5}{8}$ (E) $x > \dfrac{3}{5}$

8. The numerator and denominator of a fraction are integers differing by 16. If the value of the fraction is more than $\frac{5}{9}$ but less than $\frac{4}{7}$, then the numerator is

(A) 19 (B) 23 (C) 17 (D) 29 (E) 21

9. Civic Town has 500 voters. Everyone votes on two issues. The first issue receives 375 votes in favour and the second receives 275 votes in favour. If 40 voters vote against both issues, then the number of voters in favour of both issues is

(A) 200 (B) 190 (C) 180 (D) 310 (E) 460

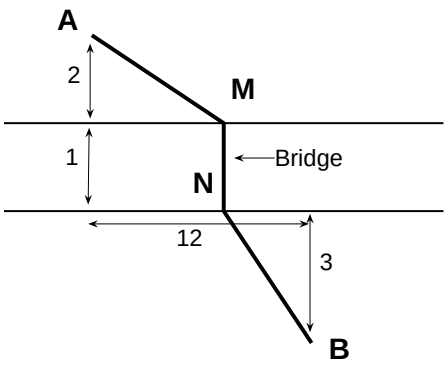
10. If x, y and z can take on only the values 1, 2, 3, 4, 5, 6, or 7, then the number of solutions of the equation $x + y + z = 12$ is

(A) 37 (B) 42 (C) 24 (D) 33 (E) 5

11. How many pairs of positive integers (a, b) are there with $a \leq b$ and $\frac{1}{6} = \frac{1}{a} + \frac{1}{b}$?

(A) 3 (B) 1 (C) 2 (D) 4 (E) 5

12. A bridge MN is to be built at right angles to the banks of a river as shown. The shortest possible path $AMNB$ connecting the two towns A and B is

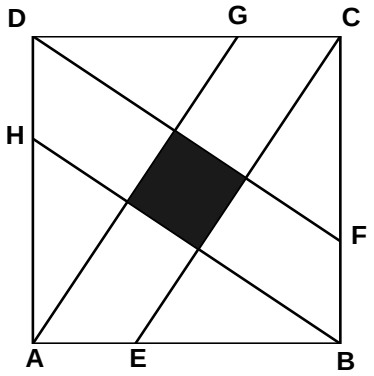


(A) 17 (B) 13 (C) 16 (D) 15 (E) 14

13. If a, b and c are three nonzero numbers and the polynomial $p(x) = x^3 - ax^2 + bx - c$ factors as $(x - a)(x - b)(x - c)$, then the value of $p(2)$ is

(A) 4 (B) 0 (C) -3 (D) 9 (E) 7

14. $ABCD$ is a square and the points E, F, G and H respectively divide the sides AB, BC, CD and DA into thirds as shown. If the area of the shaded region is 1, then the area of the square $ABCD$ is



(A) 18 (B) 6 (C) 13 (D) 4 (E) 8

15. For how many different values of $k = 1, 2, 3, \ldots$ does the k-th day of April fall on the same day of the week as the $2k$-th day of May?

(A) 1 (B) 3 (C) 2 (D) 0 (E) 4

Part C: Six marks each

16. A merchant has six barrels with capacities of 15, 16, 18, 19, 20 and 31 litres. One barrel contains wine and the other five contain oil. He keeps the barrel of wine for himself and sells the oil to two men in the ratio 1:2. The capacity of the barrel containing the wine, in litres, is,

(A) 15 (B) 31 (C) 19 (D) 18 (E) 20

17. What is the largest postage in cents that cannot be paid exactly with an unlimited supply of 6-cent and 7-cent stamps?

(A) 29 (B) 15 (C) 43 (D) 41 (E) 32

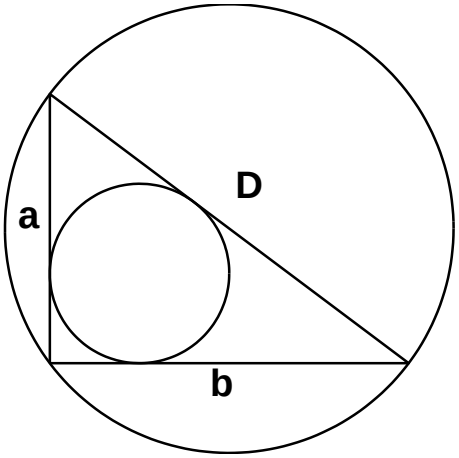
18. Suppose $f(x+1,y) = f(x,y) + y + 1$, $f(x,0) = x$ and $f(x,y) = f(y,x)$ for all x and y, then $f(12,5)$ equals

(A) 77 (B) 60 (C) 17 (D) 83 (E) 67

19. If the difference between two consecutive angles of a convex pentagon is a constant integer, then the number of possible different values of the smallest angle is

(A) 13 (B) 41 (C) 61 (D) 36 (E) 29

20. If D is the diameter of the larger circle and d the diameter of the small circle, then $D + d$ equals



(A) $a + b$ (B) $2(a + b)$ (C) $\sqrt{2}(a + b)$ (D) $2\sqrt{ab}$ (E) $\sqrt{2(a^2 + b^2)}$