THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION
Sponsored by HARMONY GOLD MINING

SECOND ROUND 2006

SENIOR SECTION: GRADES 10, 11 AND 12

17 MAY 2006

TIME: 120 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 4 marks in Part A, 5 marks in Part B and 6 marks in Part C.
   - 3.2 For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
4. You must use an HB pencil. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 120 minutes to complete the question paper.
8. Answers and solutions will be available in June at: http://www.samf.ac.za/

DO NOT TURN THE PAGE OVER UNTIL YOU ARE TOLD TO DO SO.

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0.5129; 0.9; 0.89; and 0.289 is

(A) 1.189
(B) 0.8019
(C) 1.428
(D) 1.179
(E) 1.4129

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Part A: Four marks each

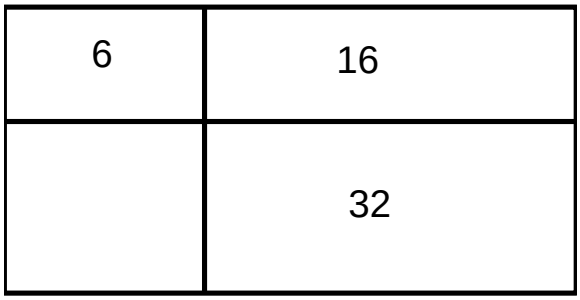
1. During a geological era, sediment is deposited in a layer 0.1 mm thick every year. The number of years for the sediment to become 7 km thick is

(A) 7 000 (B) 7 000 000 (C) 70 million (D) 700 000 (E) 700 million

2. 120% of Ellie's weight equals 75% of James' weight. The ratio of Ellie's weight to James' weight is

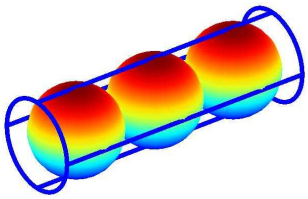
(A) $\frac{5}{8}$ (B) $\frac{5}{6}$ (C) $\frac{1}{5}$ (D) $\frac{4}{39}$ (E) $\frac{8}{13}$

3. A rectangle is divided into four rectangles as shown. The areas of three of the rectangles are given. The area of the fourth rectangle is



(A) 16 (B)10 (C) 15 (D) 14 (E) 12

4. A cylindrical container holds three tennis balls so that the balls are touching the sides and ends of the container. The ratio of the length of the container to its circumference is approximately



(A) 1:1 (B) 3:2 (C) 2:1 (D) 4:5 (E) 3:1

5. If the product of the digits of a four-digit number is 75, then the sum of the digits is

(A) 10 (B) 13 (C) 14 (D) 15 (E) impossible to determine

Part B: Five marks each

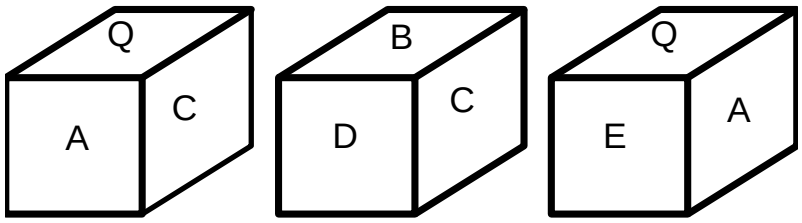
6. If b is a real number such that $b^2 = b + 1$, then which of the following is NOT true?

(A) $b^3 = b^2 + b$ (B) $b^4 = b^3 + b + 1$ (C) $b^3 = 2b + 1$ (D) $b^3 + b^2 = b + 1$
(E) $b = \dfrac{1}{b-1}$

7. If $a + b = -3$ and $ab = 4$, then $a^3 + b^3$ equals

(A) 6 (B) $3\sqrt{2}$ (C) 5 (D) $\dfrac{-3 + 2\sqrt{2}}{4}$ (E) 9

8. Three views of the same cube are shown. Each face has a different colour, labelled as in the figure. The colour of the face opposite the face with colour Q is

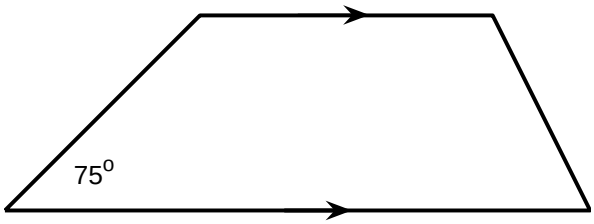


(A) A (B) B (C) C (D) D (E) E

9. $\sqrt{2 + \sqrt{3}} - \sqrt{2 - \sqrt{3}}$ equals

(A) 1 (B) $\sqrt{2}$ (C) $\dfrac{3}{2}$ (D) $\sqrt{3}$ (E) 2

10. The four angles of a trapezium have the same constant difference between them. If the smallest angle is 75° then the size of the second largest angle (in degrees) is



(A) 85 (B) 100 (C) 95 (D) 105 (E) 90

11. The mathematician Augustus De Morgan lived his entire life during the 1800's. In the last year of his life De Morgan stated: Once I was x years old in the year x^2. He was born in the year

(A) 1806 (B) 1822 (C) 1830 (D) 1851 (E) 1853

12. If one side of a triangle is 20 and the perimeter is 72, then the maximum possible area that the triangle can have is

(A) 240 (B) 200 (C) 260 (D) 460 (E) 529

13. If $\lfloor a \rfloor$ stands for the largest integer not greater than a, e.g. $\lfloor 2.7 \rfloor = 2$, then the number of solutions of the equation

$$\lfloor x \rfloor = \lfloor x + \frac{1}{2} \rfloor$$

is

(A) 0 (B) 1 (C) 2 (D) 4 (E) infinite

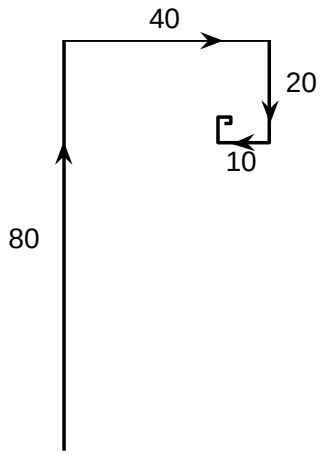
14. The average of the nine numbers

9 99 999 9999 99999 999999 9999999 99999999 999999999

is a nine-digit number M, all of whose digits are different. The number M does not contain the digit

(A) 7 (B) 1 (C) 5 (D) 0 (E) 9

15. A bug moves north, east, south, and west, and then repeats these moves indefinitely, as shown in the figure where the x-axis points east and the y-axis points north . The first leg of the journey is 80 units, and each leg is half as long as the preceding one. If the starting point is (0;0), then the ultimate destination point is



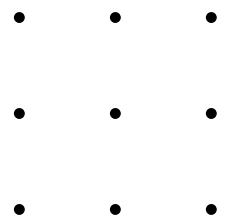
(A) $(32; 64)$ (B) (28;66) (C) (30;62) (D) (30;60) (E) (32;62)

Part C: Six marks each

16. A pentagon with area 40 has equal sides but not necessarily equal angles. The sum of the five distances from a point inside the pentagon to the sides of the pentagon is 16. The side-length of the pentagon is

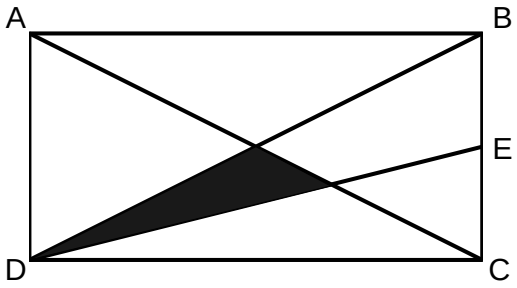
(A) $\frac{5}{2}$ (B) $\frac{5}{4}$ (C) 12 (D) 5 (E) impossible to determine

17. Nine points lie in a plane as shown. The number of triangles that can be drawn, having three of these points as vertices, is



(A) 72 (B) 84 (C) 64 (D) 76 (E) 80

18. $ABCD$ is a rectangle and E is the midpoint of BC. If the area of the rectangle is 1, then the area of the shaded region is

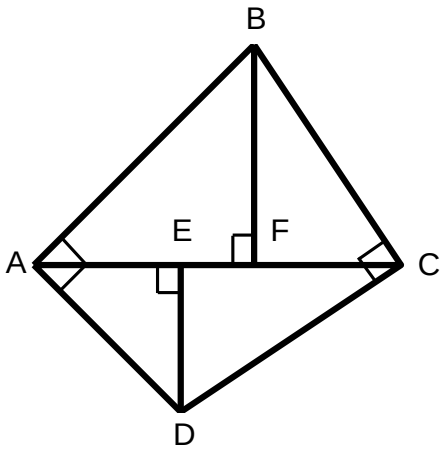


(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{1}{5}$ (D) $\frac{1}{8}$ (E) $\frac{1}{10}$

19. There are 720 different six-digit numbers that can be formed by using all of the six digits 1, 2, 3, 4, 5, and 7, for example 432751 and 731452. How many of these numbers are divisible by 11?

(A) 1 (B) 72 (C) 144 (D) 180 (E) 360

20. If $AE = 3$, $DE = 5$, and $CE = 7$ in the figure, then BF equals



(A) 3.6 (B) 4.0 (C) 4.2 (D) 4.5 (E) 5.0