SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2010 FIRST ROUND
SENIOR SECTION: GRADES 10, 11 AND 12

17 March 2010 **Time: 60 minutes** **Number of questions: 20**

Instructions

1. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
2. *Scoring rules:*
   2.1. *Each correct answer is worth 5 marks.*
   2.2. *There is no penalty for an incorrect answer or any unanswered question.*
3. *You must use an HB pencil. Rough paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
4. *Diagrams are not necessarily drawn to scale.*
5. *Indicate your answers on the sheet provided.*
6. *Start when the invigilator tells you to do so. You have 60 minutes to complete the question paper.*
7. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so
Draai die boekie om vir die Afrikaanse vraestel

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. As a decimal number 6.28% is equal to

(A) 0.0628 (B) 0.628 (C) 6.28 (D) 62.8 (E) 628

2. The value of $1 + \dfrac{1}{3 + \dfrac{1}{2}}$ is

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{6}{7}$ (E) $\frac{9}{7}$

3. The tens digit of the product $1 \times 2 \times 3 \times \cdots \times 98 \times 99$ is

(A) 0 (B) 1 (C) 2 (D) 4 (E) 9

PLEASE DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

1. A giant clock in my town counts down the days to the start of Soccer World Cup 2010. On June 11, when the tournament starts, it will show 0 days left. How many days does it show today, March 17?

(A) 86 (B) 78 (C) 82 (D) 75 (E) 89

2. The value of $\dfrac{2.010 \times 2010}{20.10 \times 201.0}$ is

(A) 0.01 (B) 0.1 (C) 1 (D) 10 (E) 100

3. Five consecutive integers add up to 50. The smallest one is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

4. On a road map, if a 7 cm length represents 56 km, how many kilometres does a 19 cm length represent?

(A) 124 (B) 152 (C) 107 (D) 160 (E) 133

5. The value of $2^{-2} + 2^{-6}$ is

(A) $\dfrac{1}{16}$ (B) $\dfrac{3}{16}$ (C) $\dfrac{3}{32}$ (D) $\dfrac{3}{64}$ (E) $\dfrac{17}{64}$

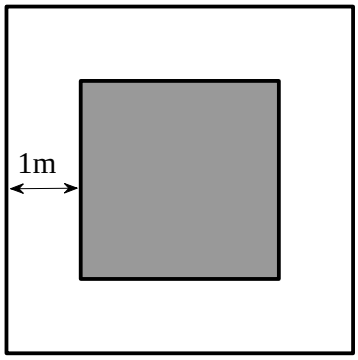
6. If Aaron Mokoena runs 12 kilometres in a 90 minute soccer match, then his average speed, in km/h, is

(A) 10 (B) 8 (C) 12 (D) 5 (E) 9

7. A collection of sheep and chickens have a total of 91 heads and legs among them. There are twice as many sheep as chickens. The number of chickens is

(A) 4 (B) 5 (C) 6 (D) 7 (E) 11

8. A square lawn has a path 1 m wide that goes around it. If the area of the path is $40\,\text{m}^2$, then the area of the lawn, in m^2, is



(A) 100 (B) 64 (C) 144 (D) 81 (E) 121

9. A profit of 25% was added to the cost price of an article to get the selling price. Later, at a discount sale the price was reduced by 20%. The percentage increase (or decrease) of the final selling price over the initial cost price was

(A) 5% increase (B) 5% decrease (C) 10% increase (D) 10% decrease (E) 0%

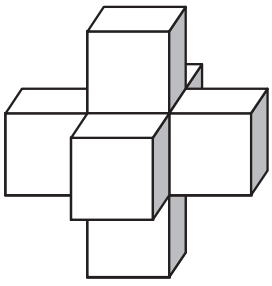
10. There are some Smarties (blue, red and yellow) in a tin. You may take out one Smartie without looking. The probability of taking out a red one is 1/4. The probability of taking out a yellow one is 1/3. The least number of blue Smarties in the tin is

(A) 5 (B) 12 (C) 3 (D) 7 (E) 4

11. If p is an odd number, which one of the following is an even number?

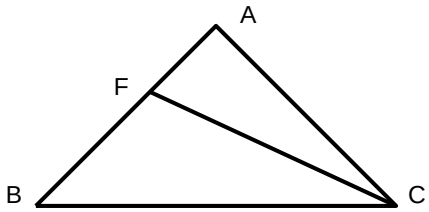
(A) $p-2$ (B) p^2 (C) p^2-2 (D) $(p-2)^2$ (E) p^2-p

12. Seven identical cubes are glued together face to face as shown in the diagram. The volume of the solid formed in this way is $189\,\text{cm}^3$. The surface area of the solid, in cm^2, is



(A) 270 (B) 300 (C) 350 (D) 360 (E) 400

13. In ΔABC, $AB = AC$, F is on AB such that $AF = FC$ and $FC = CB$. The size of $\widehat{A}$, in degrees, is

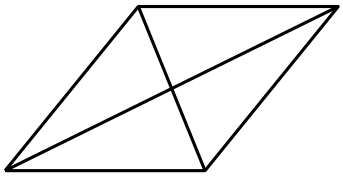


(A) 60 (B) 30 (C) 36 (D) 90 (E) 45

14. Given any two numbers whose sum is m and whose product is n. The sum of the squares of the numbers can be expressed as

(A) mn (B) $m^2 - n^2$ (C) $m^2 + 2n$ (D) $m^2 - 2n$ (E) $(m-n)^2$

15. In the rhombus one diagonal is 8 cm longer than the other diagonal. If the area of the rhombus is 24 cm^2 then the length of the longer diagonal, in cm, is



(A) 10 (B) 12 (C) 14 (D) 16 (E) 18

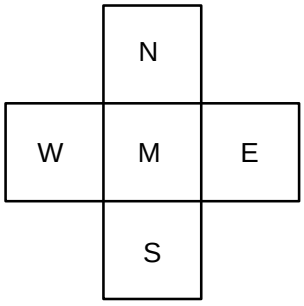
16. In the equation

$$199 + 195 + 191 + \cdots + 7 + 3 = 197 + 193 + 189 + \cdots + 5 + 1 + x$$

the value of x is

(A) 2 (B) 25 (C) 50 (D) 100 (E) 200

17. In the diagram each letter represents a different number. In how many ways can the letters represent the numbers 1, 2, 3, 4 and 5 so that the sum $W + M + E$ is equal to the sum $N + M + S$?

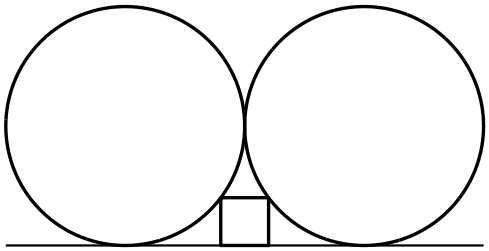


(A) 24 (B) 10 (C) 8 (D) 12 (E) 16

18. If $2^{2009} + 2^{2010}$ is divided by 5 then the remainder is

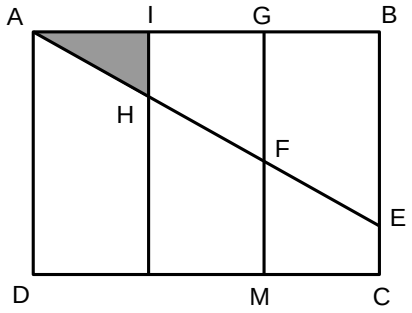
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. The diagram shows two circles each of radius one and a square of side length t. The value of t is



(A) $\frac{1}{3}$ (B) $\frac{1}{2\sqrt{2}}$ (C) $\frac{3}{8}$ (D) $\frac{\sqrt{2}}{3}$ (E) $\frac{2}{5}$

20. Rectangle ABCD is divided into three identical rectangles as shown. Line AE with E on BC is drawn such that the area of GBEF is twice the area of FECM. The ratio of the area of AIH to the area of ABCD is



(A) 1 : 24 (B) 2 : 27 (C) 2 : 45 (D) 4 : 81 (E) 1 : 21