THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION
Sponsored by HARMONY GOLD MINING

FIRST ROUND 2008

SENIOR SECTION: GRADES 10, 11 AND 12

18 MARCH 2008

TIME: 60 MINUTES

NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
   - 3.1 Each correct answer is worth 5 marks.
   - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. Paper for rough work, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.
8. Answers and solutions are available at: http://www.samf.ac.za/

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PRACTICE EXAMPLES

1. If $3x - 15 = 0$, then x is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

2. The circumference of a circle with radius 2 is

(A) π (B) 2π (C) 4π (D) 6π (E) 8π

3. The sum of the smallest and the largest of the numbers 0.5129, 0.9, 0.89, and 0.289 is

(A) 1.189
(B) 0.8019
(C) 1.428
(D) 1.179
(E) 1.4129

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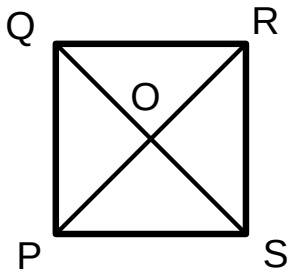
1. The value of $2008 + 8002$ is

(A) 6006 (B) 1010 (C) 10 010 (D) 2222 (E) 10 000

2. The value of $1 + \dfrac{1}{3 + \dfrac{1}{2}}$ is

(A) $\frac{6}{5}$ (B) $\frac{7}{6}$ (C) $\frac{9}{2}$ (D) $\frac{6}{7}$ (E) $\frac{9}{7}$

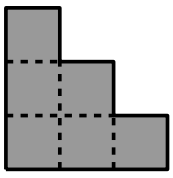
3. The diagonals of square PQRS intersect at O. Triangle SOR has area 16. The length of PQ is



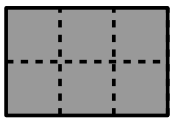
(A) 2 (B) 4 (C) 6 (D) 8 (E) 16

4. All the shapes have the same area. Which shape has the smallest perimeter?

(A)

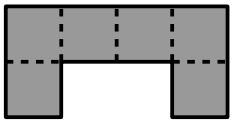


(B)

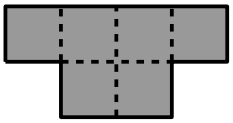


(C)

(D)



(E)



5. If $f(x) = 2x - 1$, then $f(f(f(2)))$ equals

(A) 2 (B) 4 (C) 6 (D) 7 (E) 9

6. Which one of the following is an even number?

(A) $2007^3 + 4$ (B) $2008^3 + 5$ (C) $2009^3 + 6$ (D) $2009^3 + 7$ (E) $2010^3 + 9$

7. Each tyre of car A has circumference 200 cm. Each tyre of car B has circumference 250 cm. On a journey of 20 km, the difference in the number of revolutions made by a wheel of car A and a wheel of car B equals

(A) 800 (B) 1000 (C) 2000 (D) 4000 (E) 10 000

8. Tebatso defined a new way of combining two positive integers m and n:

$$m \diamondsuit n = \frac{\text{the lowest common multiple of } m \text{ and } n}{\text{the highest common factor of } m \text{ and } n}.$$

For example $12 \diamondsuit 30 = \dfrac{60}{6} = 10$.

The value of $(6 \diamondsuit 4) \diamondsuit 16$ is

(A) 24 (B) 18 (C) 12 (D) 6 (E) 1

9. The average mark for 100 learners on a mathematics test is 39 %. The average mark for the learners who passed the test is 60 %. The average mark for the learners who failed the test is 30 %. The number of learners who passed is

(A) 30 (B) 40 (C) 50 (D) 60 (E) 70

10. Two-thirds of the members of a committee use three-quarters of the chairs in a room. What is the smallest number of members that the committee can consist of?

(A) 6 (B) 8 (C) 9 (D) 12 (E) 15

11. The largest value of n such that 3^n divides into $1 \times 3 \times 5 \times 7 \times 9 \times \cdots \times 31$ without remainder is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

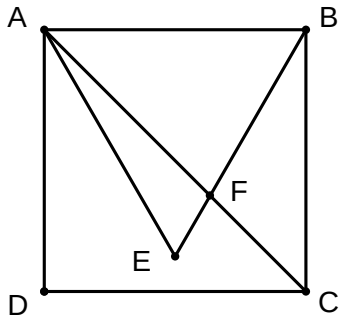
12. The line containing the points $(2, a)$ and $(4, b)$ has slope (gradient) equal to -2. The slope of the line containing points $(2, -a)$ and $(4, -b)$ equals

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

13. A dice is thrown twice. The first throw determines the tens digit and the second throw the ones digit of a two-digit number. The probability that this two-digit number is a perfect square equals

(A) $\frac{1}{12}$ (B) $\frac{7}{36}$ (C) $\frac{1}{18}$ (D) $\frac{1}{5}$ (E) $\frac{1}{9}$

14. If ABCD is a square and ABE is an equilateral triangle, then angle BFC, measured in degrees, equals



(A) 100 (B) 120 (C) 135 (D) 115 (E) 105

15. How many digits does $625^2 \times 32^2 \times 7$ have when multiplied out?

(A) 13 (B) 10 (C) 12 (D) 14 (E) 16

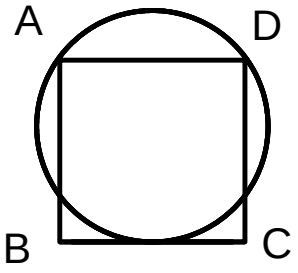
16. For how many whole numbers between 100 and 999 does the product of the ones digit and tens digit equal the hundreds digit?

(A) 20 (B) 23 (C) 21 (D) 25 (E) 26

17. How many real solutions does $x + \sqrt{x^2 + \sqrt{x^3 + 1}} = 1$ have?

(A) 1 (B) 2 (C) 3 (D) none (E) infinitely many

18. A circle passes through vertices A and D and touches side BC of a square. If the square has side length 2, then the radius of the circle is



(A) $\frac{5}{4}$ (B) $\frac{4}{5}$ (C) 1 (D) $\frac{5}{2}$ (E) $\frac{3}{2}$

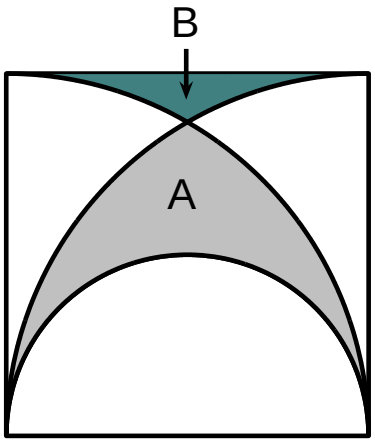
19. The function $f(x)$ satisfies the equation

$$f(1 - x) + 2f(x) = 3x$$

for all real x. The value of $f(0)$ is

(A) -2 (B) -1 (C) 0 (D) 1 (E) 2

20. The diagram shows a semicircle and two quarter circles inscribed in a square of side length 2. The difference between the area of the shaded region A and the area of the shaded region B equals



(A) $\frac{3}{2}\pi - 4$ (B) $\frac{1}{3}\pi - \frac{1}{3}$ (C) $\frac{3}{2} - \frac{1}{4}\pi$ (D) $4 - \pi$ (E) $\frac{1}{4}\pi$