



THE HARMONY SOUTH AFRICAN  
MATHEMATICS OLYMPIAD

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Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION  
Sponsored by HARMONY GOLD MINING

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FIRST ROUND 2006  
SENIOR SECTION: GRADES 10, 11 AND 12  
23 MARCH 2006  
TIME: 60 MINUTES  
NUMBER OF QUESTIONS: 20

**Instructions:**

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice answer paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Scoring rules:
  - 3.1 Each correct answer is worth 5 marks.
  - 3.2 There is no penalty for an incorrect answer or any unanswered questions.
4. Rough paper, ruler and rubber are permitted. **Calculators and geometry instruments are not permitted.**
5. Diagrams are not necessarily drawn to scale.
6. Indicate your answers on the sheet provided.
7. Start when the invigilator tells you to. You have 60 minutes to complete the question paper.
8. Answers and solutions are available at: <http://www.samf.ac.za/>

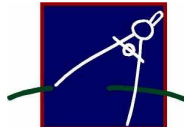
**DO NOT TURN THE PAGE OVER UNTIL YOU  
ARE TOLD TO DO SO.**

**DRAAI DIE BOEKIE OM VIR AFRIKAANS**

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## PRACTICE EXAMPLES

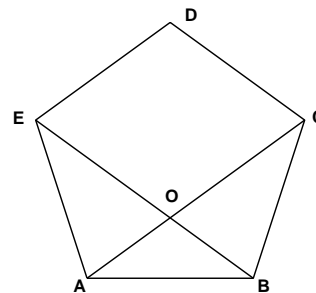
1. If  $3x - 15 = 0$ , then  $x$  is equal to  
(A) 2                      (B) 3                      (C) 4                      (D) 5                      (E) 6
2. The circumference of a circle with radius 2 is  
(A)  $\pi$                       (B)  $2\pi$                       (C)  $4\pi$                       (D)  $6\pi$                       (E)  $8\pi$
3. The sum of the smallest and the largest of the numbers 0.5129; 0.9; 0.89; and 0.289 is  
(A) 1.189  
(B) 0.8019  
(C) 1.428  
(D) 1.179  
(E) 1.4129

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1. An approximate value for  $\frac{302.476 \times 0.040328}{5.96247}$  is  
(A) 2            (B) 10            (C) 200            (D) 20 000            (E) 1 000 000
2. Rectangle  $ABCD$  has sides  $AB$  and  $BC$  in the ratio 3 : 1. If the diagonal  $AC$  is 5, then the area of the rectangle is  
(A) 9            (B)  $\frac{15}{2}$             (C) 8            (D) 10            (E)  $\frac{20}{3}$
3. If  $x^2 = x + 3$ , then  $x^3$  equals  
(A)  $x^2 + 9$             (B)  $x^2 + 3x + 3$             (C)  $4x + 3$             (D)  $x + 3$             (E)  $x^2 + 3$
4.  $2006^2 - 2007 \times 2005 + 2008 \times 2004 - 2009 \times 2003$  equals  
(A) -2            (B) 2            (C) 6            (D) 2000            (E) 2006
5. The next number in the pattern 2; 3; 6; 15; 42; ... is  
(A) 111            (B) 123            (C) 135            (D) 148            (E) 162
6. The last (units) digit of the number  $333^{444}$  is  
(A) 1            (B) 5            (C) 3            (D) 7            (E) 9
7. If  $\frac{2x - 3y}{x + 2y} = 3$ , then the numerical value of  $\frac{2x + y}{3x + 10y}$  is  
(A)  $\frac{19}{28}$             (B)  $\frac{1}{2}$             (C)  $\frac{2}{3}$             (D)  $\frac{7}{9}$             (E) 1
8.  $4^{n+1} + 4^{n+2}$  equals  
(A)  $8^{2n+3}$             (B)  $4^{2n+3}$             (C)  $5 \times 2^{2n+2}$             (D)  $5 \times 4^{2n+3}$             (E)  $5 \times 4^{n+3}$

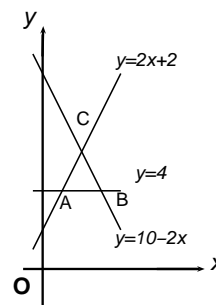
9. If  $f(1) = 0$  and  $f(n) = f(n - 1) + 2n - 1$  for  $n > 0$ , then the value of  $f(3)$  is
- (A) 8                      (B) 9                      (C) 7                      (D) 5                      (E) 2

10. If  $ABCDE$  is a regular pentagon and  $EB$  and  $AC$  intersect at  $O$ , then the size of angle  $\widehat{EOC}$  in degrees is



- (A) 100                      (B) 108                      (C) 135                      (D) 96                      (E) 90

11. The area of triangle  $ABC$  in the figure is

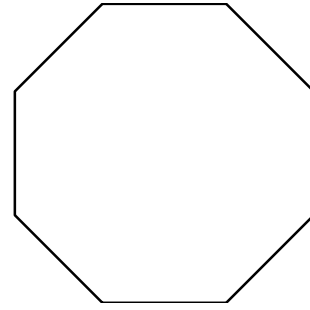


- (A) 2                      (B) 3                      (C) 4                      (D) 3.5                      (E) 2.5

12. Aneesa, Bongi and Carol are wearing dresses and shoes that are green, black or yellow. No two dresses or pairs of shoes are the same colour. Aneesa has yellow shoes. Bongi does not have a black dress or black shoes and only Carol has the same colour dress and shoes. Bongi has

- (A) A green dress and yellow shoes  
 (B) A black dress and green shoes  
 (C) A green dress and green shoes  
 (D) A green dress and black shoes  
 (E) A yellow dress and green shoes

13. The regular octagon has area 500. The best estimate of the length of one side is



- (A) 10                      (B) 15                      (C) 12                      (D) 5                      (E) 20

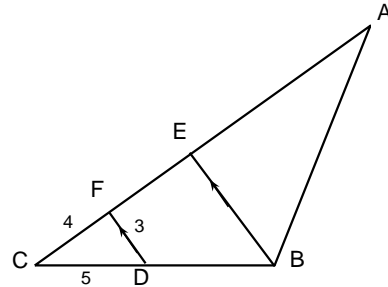
14. If  $a_n = \frac{1}{n} - \frac{1}{n+1}$ , then  $a_1 + a_2 + \dots + a_{100}$  equals

- (A)  $\frac{99}{100}$                       (B)  $\frac{100}{101}$                       (C) 1                      (D)  $\frac{101}{100}$                       (E)  $\frac{100}{99}$

15. A motorist covers 177.5 km in 2 hours. On the open road he averages 105 km/h and 40 km/h in urban areas. How many minutes did he take to pass through the urban areas?

- (A) 60                      (B) 45                      (C) 30                      (D) 15                      (E) 75

16. In triangle  $ABC$ ,  $E$  is the midpoint of  $AC$  and  $D$  is the midpoint of  $CB$ . If  $DF$  is parallel to  $BE$ , then the length of side  $AB$  is



- (A) 8                      (B) 10                      (C) 14                      (D) 13                      (E) 11

17. A bag contains 6 blue balls, 8 yellow balls and 2 pink balls. Siphon takes balls from the bag without looking at them. The least number of balls that he must remove in order to ensure that he has three of the same colour is

- (A) 3                      (B) 5                      (C) 7                      (D) 9                      (E) 11

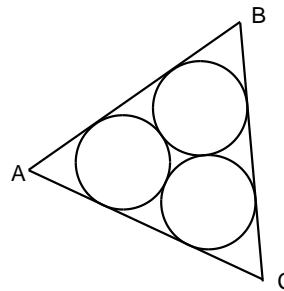
18. If  $x = 1 + 2^p$  and  $y = 1 + 2^{-p}$  then  $y$  equals

- (A)  $\frac{x+1}{x-1}$       (B)  $\frac{x+2}{x+1}$       (C)  $\frac{x}{x-1}$       (D)  $2-x$       (E)  $\frac{x-2}{x+1}$

19. If  $x^2 + 6x + y^2 - 4y + 13 = 0$ , then  $x + y$  is

- (A) -1      (B) -2      (C) 0      (D) 1      (E) impossible to determine

20. Three touching circles, each with radius 1, are inscribed in triangle  $ABC$ . The length of side  $AB$  of the triangle is



- (A) 6      (B)  $3\sqrt{3}$       (C)  $\frac{\sqrt{3}}{2}$       (D)  $1 + \sqrt{3}$       (E)  $2 + 2\sqrt{3}$
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