HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION in collaboration with the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN KUNS, AMESA and SAMS. Sponsored by HARMONY GOLD MINING.

Third Round 2007
Junior Section: Grades 8 and 9
Date: 6 September 2007

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- The neatness in your presentation of the solutions may be taken into account.
- The time allocated is 4 hours.
- No calculator of any form may be used.
- Answers and solutions are available at:

www.samf.ac.za

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UNTIL YOU ARE TOLD TO DO SO.

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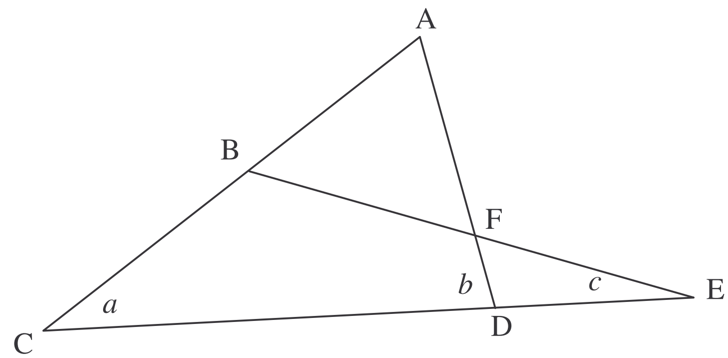
Question 1

Find the 2007th letter in the sequence:

ABCDEFGFEDCBAABCDEFGFEDCBAABCD……….. **(4)**

Question 2

Given that AB = AF, find the relationship between a, b and c.



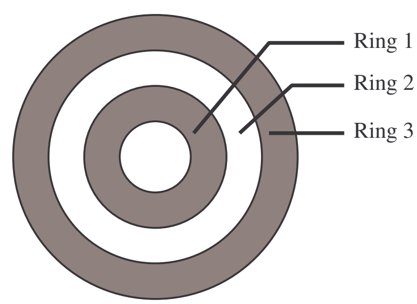
(4)

Question 3

The figure shows four circles with the same centre and radii of 1, 2, 3 and 4 respectively.

(a) Calculate the area of ring 3 in terms of π.

(b) If there are 2007 rings, find the area of the outside ring in terms of π.



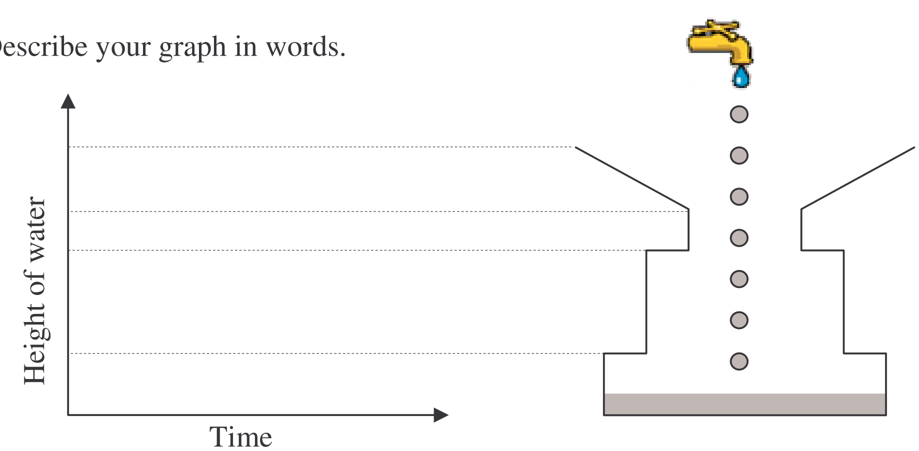
(6)

Question 4

Water drips at a constant rate into a container, as shown in the diagram.

(a) Draw a graph of the height of the water in the container against time, until the container is full.

(b) Describe your graph in words.



(6)

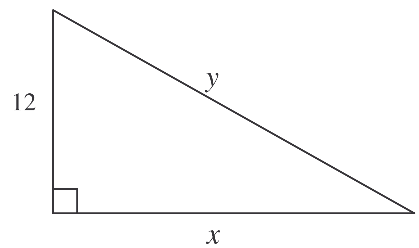
Question 5

The sum of the factors of 120 is 360. Find the sum of the reciprocals of the factors of 120.

(6)

Question 6

In the right angled triangle below, for which positive integer value(s) of x is the hypotenuse y an integer?

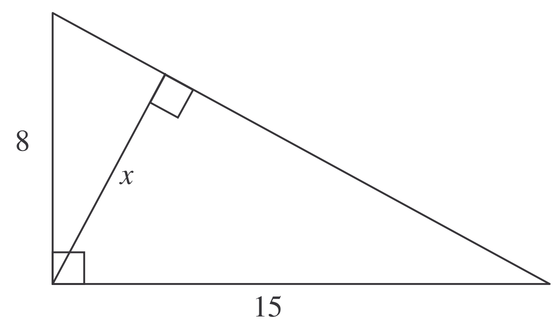


(6)

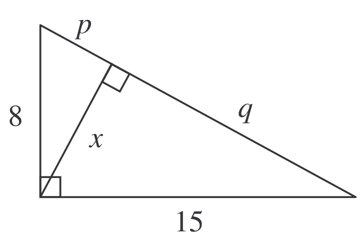
Question 7

The following question was given to a class for homework:

"In the diagram, find the length of x "



Saskia did it as follows:



$$p^2 + x^2 = 64 \qquad (1)$$

$$q^2 + x^2 = 225 \qquad (2)$$

$$8^2 + 15^2 = (p+q)^2$$

$$\therefore p + q = 17 \qquad (3)$$

$$(2)-(1) \qquad q^2 - p^2 = 161$$

$$(q-p)(q+p) = 161$$

$$\therefore q - p = \frac{161}{17} \qquad (4)$$

$$(3)+(4): \qquad 2q = \frac{450}{17}$$

$$\therefore q = \frac{450}{34}$$

$$\text{From } (2): \qquad 225 - (\frac{450}{34})^2 = x^2$$

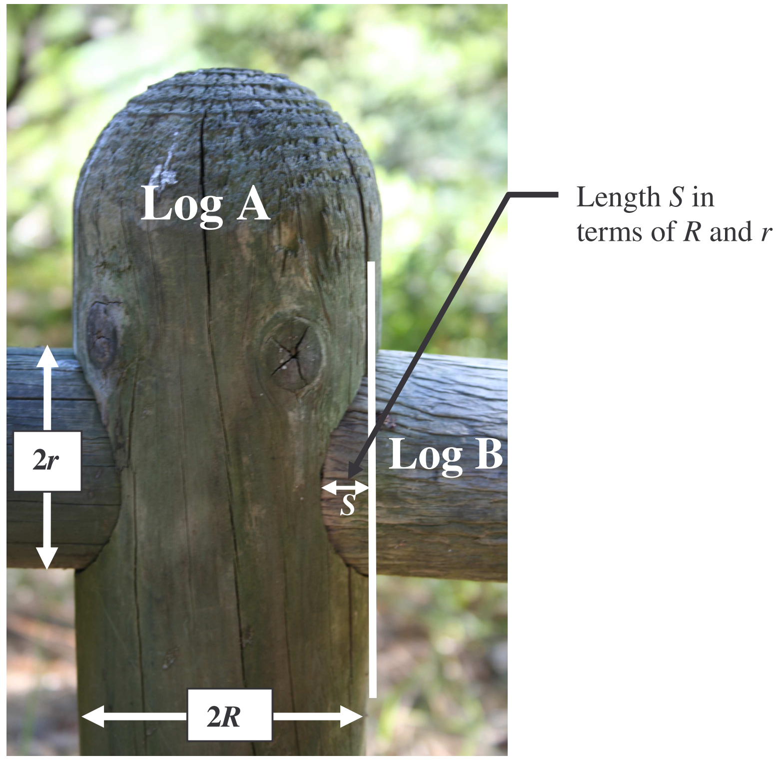
$$\therefore x = \frac{120}{17}$$

Anil did it in a far quicker and elegant way. Find Anil's solution to the problem.

(6)

Question 8

In the diagram, the log A has radius R. A hole of radius r is drilled through the centre of log A at right angles to the axis.
Another log B of radius r passes through the hole.
Find the length S in terms of R and r.



(6)

Question 9

Two rugby teams (thirty players) have been selected to play for their school. Five of the players speak Sotho, Afrikaans and English. Nine of them speak only Sotho and English. Twenty speak Afrikaans, of which twelve also speak Sotho. Eighteen speak English. No one speaks only Sotho. How many players speak only English? **(8)**

Question 10

Molly is in Grade 3 and in a test on fractions she wrote the following:

$$\frac{1}{2}+\frac{1}{3}=\frac{2}{5}$$

(a) If Molly used the same method, what would her answer to the following sum be?

$$\frac{2}{3} + \frac{5}{4} =$$

(b) Prove that there are no integer values for a and b such that

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{a+b}$$

(c) There are however integer values for a, b, c and d such that

$$\frac{a}{b} + \frac{c}{d} = \frac{a+c}{b+d}$$

Find a set of values for a, b, c and d $(a \neq b \neq c \neq d)$ that make the equation above true. **(8)**

Question 11

(a) Find the value of $1 + 2(1 + 2(1 + 2(1 + 2)))$

(b) Find the value of $1 + 2(1 + 2(1 + \ldots\ldots \underbrace{) \ldots))))}_{\text{2007 brackets}}$

(8)

Question 12

In the game ***"Move out"***, two players take it in turns to move any one of the three counters on the board any number of squares to the right, beginning from their 'start' blocks, until all three counters are in the 'end' blocks. The last player to move a counter loses.

Start 1	2	3	4	5	6	A 7	8	9	**End** 10
Start 1	2	3	4	5	6	7	B 8	9	**End** 10
Start 1	2	3	4	5	6	7	C 8	9	**End** 10

(a) If it was your turn to play in the game above, which counter would you move and to which position, to guarantee you a win.

(b) Explain your answer.

(8)

Question 13

Observe the following patterns:

$$23 \times 28 = 20 \times 31 + 3 \times 8 = 620 + 24 = 644$$
$$51 \times 59 = 50 \times 60 + 1 \times 9 = 3000 + 9 = 3009$$

(a) Calculate 65×69 in the same way.

(b) Prove that the pattern holds for any two 2-digit numbers that have the same tens-digit.

(8)

Question 14

In a computer game, you have to score the largest possible number of points. You score 9 points each time you find a jewel and 5 points each time you find a sword. There is no limit to the number of points you can score. Of course it is impossible to score 6 or 11 points.

(a) What is the largest number of points impossible to score?

(b) Prove that this is in fact the largest number.

(8)

Question 15

(a) Is the following divisible by 3?

$$2007^3 - 2006^3 + 2005^3 - 2004^3 \ldots\ldots\ldots - 2^3 + 1^3$$

(b) Prove your answer.

(8)

Total: 100

THE END