HARMONY

HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION in collaboration with the SUID-AFRIKAANSE AKADEMIE VIR WETENSKAP EN KUNS, AMESA and SAMS. Sponsored by HARMONY GOLD MINING.

Third Round 2006
Junior Section: Grades 8 and 9
Date: 7 September 2006

Instructions

- Answer all the questions.
- All working details and explanations must be shown. Answers alone will not be awarded full marks.
- This paper consists of 15 questions for a total of 100 marks as indicated.
- The neatness in your presentation of the solutions may be taken into account.
- The time allocated is 4 hours.
- No calculator of any form may be used.
- Answers and solutions are available at:

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UNTIL YOU ARE TOLD TO DO SO.

PRIVATE BAG X11, ARCADIA, 0007 TEL: (012) 392-9323 FAX: (012) 320-1950
E-mail:ellie@samf.ac.za

1. Write the number 1 000 000 as the product of two positive integers neither of which has any zeros in it. (4)

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2. Find the product of 968 880 726 456 484 032 and 125. (4)

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3. (a) Find five positive integers, not necessarily different, whose sum is equal to their product.

(b) Show that the problem in (a) has at least three solutions. (6)

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4. Find all integer solutions of the equation: $\frac{1}{x}+\frac{1}{y}=\frac{1}{2}$. (6)

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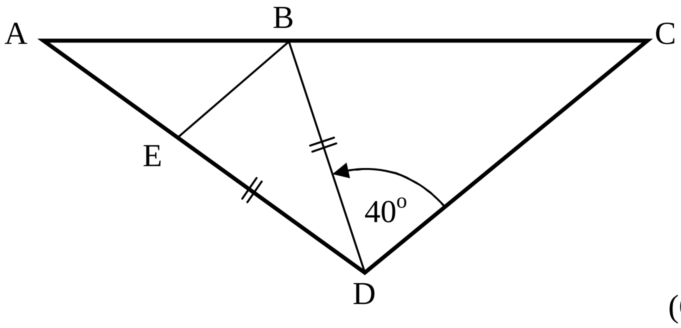
5. In the figure below, AD = DC; ED = BD and $B\hat{D}C = 40^\circ$.



Find the size of $A\hat{B}E$. (6)

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6. Evaluate the following sum:

$$\frac{1}{2}+\left(\frac{1}{3}+\frac{2}{3}\right)+\left(\frac{1}{4}+\frac{2}{4}+\frac{3}{4}\right)+\left(\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}\right)+\cdots+\left(\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\cdots+\frac{99}{100}\right).$$

(6)

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7. Write the numbers

$$a=\sqrt{0,16};\qquad b=\sqrt[3]{0,063};\qquad c=\sqrt[5]{0,01025}\quad\text{and}\quad d=(0,2)^2$$

in increasing order. (6)

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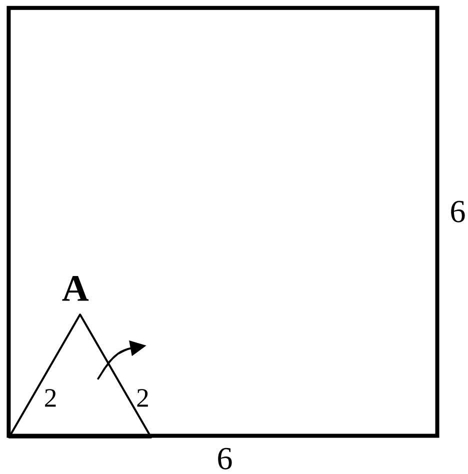
8. An equilateral triangle of side 2 is rolled along the inside of a square of side 6 until the vertex A returns to its original position, as shown. Determine the length of the path the point A has travelled. Leave your answer in terms of π. (6)

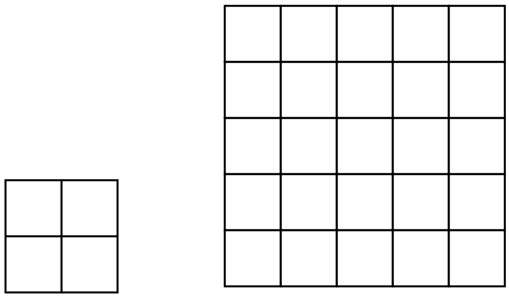


9. The letters *M, A, T, H, S* denote positive **real numbers** such that $M \times A = 12$, $T \times H = 30$, $A \times H = 24$, $A \times T = 20$ and $H \times S = 42$. Find the value of $M \times A \times T \times H \times S$.

(6)

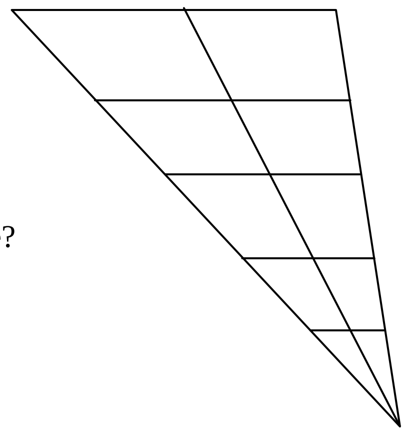
10. A string of 2006 digits begins with the digit 6. Any number formed by two consecutive digits is divisible by 17 or 23.
Write down the last five digits. (6)

11. Show that it is possible to fill in a 5 by 5 table using numbers such that the sum of all of them is positive, but the sum of any four numbers forming a 2 by 2 block is negative.



(6)

12. The figure alongside has 3 oblique lines and 5 horizontal lines and contains 15 triangles of different shapes and sizes.
How many triangles (of any size) does a similar figure with ***s*** oblique lines and ***h*** horizontal lines have?



Copy this table into your answer book and fill in as many numbers as you need to find the number of triangles with ***s*** oblique lines and ***h*** horizontal lines.

Number of oblique lines	**Number of horizontal lines**	**Number of triangles**
3	5	15
s	***h***	**?**

(8)

13. In the alphanumeric puzzle $OAK + OAK + \cdots + OAK = PINE$, equal letters correspond to equal digits and different letters correspond to different digits, and O does not stand for 0.
Find the maximum possible number of *OAK's* that will satisfy the equation and explain why the number you have found is in fact a maximum. (8)

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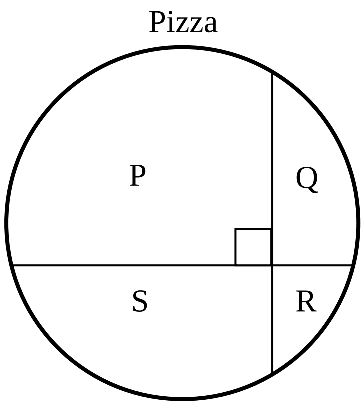
14. Andy and Bongi order pizza. The pizza is divided into four pieces with two straight perpendicular cuts that do not pass through the centre. Andy can choose either P and R or Q and S.
Which pieces should Andy choose in order to get more pizza than Bongi? Justify your answer.



(8)

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15. (a) Find the sum of the digits of the product 99999×66666. (2)

(b) Find an expression in $\boldsymbol{n}$ and/or $\boldsymbol{k}$ for the sum of the digits of the product below and prove that it holds for all $\boldsymbol{n}$ and/or $\boldsymbol{k}$:

$$\underbrace{999\ldots9}_{n\ 9's} \times \underbrace{kkk\ldots k}_{n\ k's} \qquad \text{(where } n, k \in \mathbb{N} \text{ and } 0 < k < 10\text{)}$$

(8)

(c) Find an expression for the sum of the digits of

$$\underbrace{999\ldots9}_{n\ 9's} \times \underbrace{kkk\ldots k}_{m\ k's} \qquad \text{(where } n, m, k \in \mathbb{N} \text{ and } 0 < k < 10\text{)}$$

(4)

TOTAL: 100

THE END