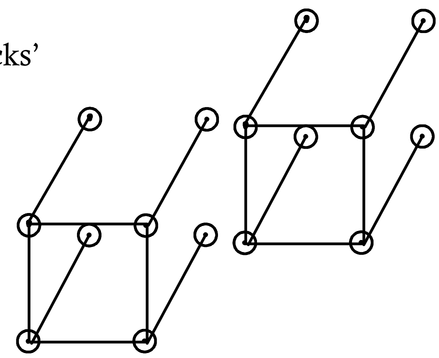
SOUTH AFRICAN MATHEMATICS OLYMPIAD 2011
Junior Round 2
Solutions

1. **A** All the fractions lie between 1 and 2 and have the form $1+\frac{10}{n}$; so the one nearest 2 is the largest one, which is the one with n smallest.

2. **E** The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40: the sum of these is 90, so when Johan got a total of 70 he must have left out 20

3. **A** The structure can be seen as made up of 40 'blocks' joined together, with one cap at the end, which requires $40 \times 8 + 4 = 324$ rods



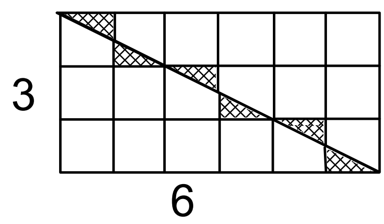
4. **E** 20^{11} is $2^{11} \times 10^{11}$, and 2^{11} is 2 048. So the result is $2\,048 \times 10^{11}$, which is $2{,}048 \times 10^{14}$ and therefore has 15 digits.

5. **B** After the first gift, Rebecca has kept 90% of what she started with. After the second gift she has retained 80% of that, so she is left with $0{,}80 \times 0{,}90 = 0{,}72$, i.e. 72%, of what she started with.

6. **E** Since F = 2C, the formula gives $C=\frac{5}{9}(2C-32)$. Hence 9C = 5(2C – 32) and so 9C = 10C – 160, which means that C = 160.

7. **D** Time is distance ÷ speed; so it takes Boris $\frac{4}{10}$ of an hour to reach O. In that time Otto moves a distance of $\frac{4}{10}\times 6$ = 2,4 km, and he is therefore then 8 + 2,4 = 10,4 km from O

8. **C** Completing the rectangle of which the required triangle is half, and looking at the shaded triangles which are congruent to each other in pairs, we see that whenever one tile is cut it provides two of the triangular pieces we need. So three tiles have to be cut, and 6 tiles are used uncut, so we need at least 9 tiles altogether



9. **B** The product is $\frac{\cancel{3}}{2}\times\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{5}}{\cancel{4}}\times\ldots\times\frac{20}{\cancel{19}}$, which cancels down to $\frac{20}{2}$ = 10

10. **A** Δ APB must be half the rectangle, so Δ ADP + Δ BPC is half the rectangle, and that means Δ BPC is ½ - 1/5 = 5/10 – 2/10 = 3/10 of the rectangle.

11. **B** The numbers used must come from the sequence 6; 13; 20; 27; 34; 41; .... While 6; 13; 41; 55 contains four numbers that have no common factors, a smaller sum is achieved with 13; 20; 27; 41

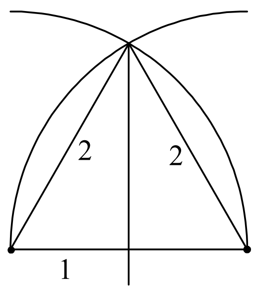
12. **C** 366 divided by 7 is 52 with remainder 2; so there are only two days of which there can be 53, and since there are 7 days in a week, the probability that one of these is Sunday is 2/7

13. **D** We see four odd numbers and two even ones; on the other side of the even numbers must be odd numbers, so we know of 4 visible and 2 hidden odd numbers. We cannot tell what is on the reverse of the odd numbers, so there might be another four hidden odd numbers.

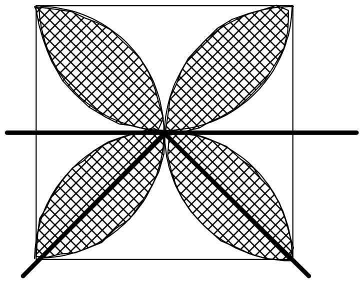
14. **B** The four removed squares have total area $1^2 + 2^2 + 3^2 + 6^2 = 50$, so the original piece of paper had area 100 cm^2 and therefore side 10 cm. The perimeter of the original sheet has not changed: every time edges are removed they are replaced by exactly equal ones. So the perimeter of the remaining shape is $4 \times 10 = 40$ cm.

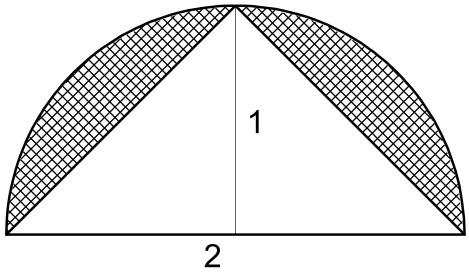
15. **E** Let the number of girls be $3a$, so that the number of boys is $2a$. Then $\frac{2a+5}{3a} = \frac{7}{10}$. This equation gives $20a + 50 = 21a$, so that $a = 50$ and the number of girls is $3a = 150$.

16. **D** The triangle formed when the centres of the circles are joined to one of the points of intersection is equilateral, with sides equal to the radii of the circles. The line joining the points of intersection bisects this equilateral triangle, and by Pythagoras the height of the triangle is $\sqrt{3}$. By symmetry the length of the common chord is twice that.



17. **B**





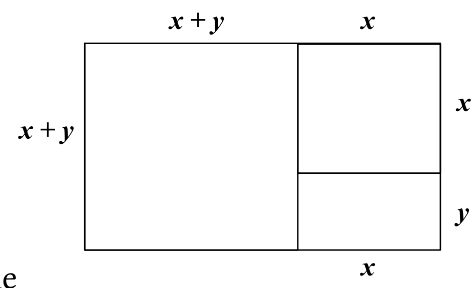
Considering only a part of the original diagram, we can see some of the shaded area as that part of a semicircle which is outside a right-angled triangle. The shaded area in the second diagram is $\frac{1}{2}\pi.1^2 - \frac{1}{2}.2.1$, while the whole four-leaf shape must have area 4 times this, which is $2\pi - 4$

OR

The semicircles, with radius 1, have a combined total area of $4 \times \frac{1}{2}\pi(1)^2 = 2\pi$. The square has an area of $2^2 = 4$. The overlap is exactly the amount by which the area of the semicircles exceeds that of the square, i.e. $2\pi - 4$ units.

18. **C** Let a three-digit number be 100X + 10Y + Z. When it is reversed it becomes 100Z + 10Y + X, and (assuming Z < X) the subtraction gives 99X – 99Z = 99(X – Z). Thus no matter what the starting number, the result of the subtraction must be divisible by 99, and only option C satisfies this condition.

19. **A** If we let the sides of the smallest rectangle have lengths x and y, the other lengths have to be as shown. Now the relationship between the perimeters becomes $2(x + y) = \frac{3}{8}.2(x + y + x + y + x)$, which gives $16(x + y) = 6(3x + 2y)$. Hence $16x + 16y = 18x + 12y$, so $2y = x$, and now the ratio of the sides of the large rectangle is $x + y : (2x + y) = 3y : 5y = 3 : 5$.



20. **B** There are 5 × 4 × 3 = 60 different possible three-digit numbers. Imagine all these possibilities written in a list, one above the next. Then the same digits (1; 3; 5; 7; 9) appear in the three columns, each with equal frequency, so the digits in each of the three columns when the 60 numbers are added together have sum 12(1+3+5+7+9) = 300. The sum of all 60 numbers is therefore 300(100 + 10 + 1) = 33 300.