SAMO 2010 – Junior Second Round SOLUTIONS

Part A: (Each correct answer is worth 4 marks)

1. D. The last two digits must be at least 05 since the double of it must not start with zero. The last two digits must be at most 49 so that the double of that uses only two digits. So the last two digits can be anything from 05 up to 49, and then for each of these the first two digits are determined. This gives 45 possibilities

2. D. ΔABC is isosceles, with $\hat{B} = 108°$, the interior angle of a regular pentagon. Then $2x + 108° = 180°$, so $x = 36°$

3. C. The volume of the tank is $200 \times 200 \times 400$ cm^3, but water is pouring in at the rate of 5 000 cm^3 per second. The time taken to fill the tank is then $\frac{200 \times 200 \times 400}{5000} = \frac{16000}{5} = 3200$ seconds, and dividing by 60 shows this is a little over 53 minutes.

4. D. If the number of CDs that Jane has today is Y, then once John has given four of his CDs to her, he will have $X - 4$ and she will have $Y + 4$. So we know that $Y + 4 = 2(X - 4)$, and therefore that $Y = 2(X - 4) - 4 = 2X - 12 = 2(X - 6)$

5. D. The first equation can be rearranged as $a + d = c - b$, and the second equation can be rearranged as $a + d = b - c$. Adding these shows $2(a + d) = 0$.

Part B: (Each correct answer is worth 5 marks)

6. C. If Bernard's age now is x, then Charlie's age is $x + 4$ and therefore Arthur's age is $\frac{1}{2}(x + 4)$. The sum of their ages in 6 years' time will be $(x + 6) + (x + 4 + 6) + (\frac{1}{2}(x + 4) + 6)$, which simplifies to $\frac{5}{2}x + 24$, and if this is 69 then $\frac{5}{2}x = 45$ and so x is 18

7. C. The numbers leaving a remainder of 1 on division by 21 are 1, 22, 43, ... and there are 47 of them. The remainders when these numbers are divided by 35 are 1, 22, 8, 29,15,1, 22, … so every fifth one leaves a remainder of 1. There are therefore 9 of them after the initial one, so ten of them altogether.

8. A. Out of every 40 pupils, 25 (i.e. $\frac{5}{5+3}$ of them) are seniors and 15 are juniors. Out of every 15 juniors, 9 (i.e. $\frac{3}{2+3}$ of them) are boys and 6 are girls, while for every 25 seniors 10 are boys and 15 are girls. So of every 40 pupils, 9 + 10 = 19 are boys and therefore 21 are girls, and so the ratio of boys to girls in the school is 19 : 21

9. B. If Dave's speed is x km/h, and T is the time that he takes, then xT is the distance he travels. John's distance must be $(x + 10) \times T.\frac{1}{3}$, and these are equal so $xT = (x + 10)\frac{T}{3}$. Cancelling T and multiplying out gives $3x = x + 10$, so $x = 5$.

10. B. $4^x \times 6^y = 48^{12}$, so $2^{2x} \times 2^y \times 3^y = 2^{2x+y} \times 3^y = (2^4.3)^{12} = 2^{48}.3^{12}$. This requires $2x + y = 48$ and also $y = 12$, which means that $x = 18$ and finally we have $x + y = 30$.

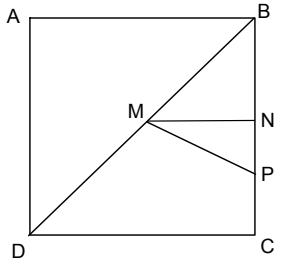
11. E. Considering the terms in pairs up until almost the end shows the sum must be $-3-7-11-\ldots.-47+625$.
This is $(-4.1+1)+(-4.2+1)+(-4.3+1)+\ldots.+(-4.12+1)+625$, which is $-4(1+2+3+\ldots+12)+1\times12+625$, i.e. $-4.(\frac{1}{2})12.13+12+625$, and that is $-26.12+1.12+625=625-25.12=625-300=325$.

OR: since the terms are all squares, pair them thus:
$1+(9-4)+(25-16)+\ldots(625-576)$
$=1+(3-2)(3+2)+(5-4)(5+4)+\ldots+(25-24)(25+24)$
$=1+3+2+5+4+\ldots+25+24$
$=1+2+3+\ldots+25=\frac{1}{2}.25.26=325$.

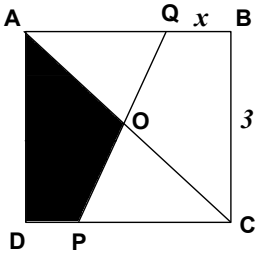
OR: if we consider the numbers in pairs from the second one onwards we get 1 + 5 + 9 + 13 + 17 + . . . + 25 + . . . + 45 + 49. There are 13 of these numbers and the middle one is 25. Notice that 1 + 49 = 50, 5 + 45 = 50, etc. There are 6 such pairs and also the middle number 25, so the sum is $6 \times 50 + 25 = 325$.

12. B. The stated conditions mean that the first digit can be any one of the five digits 1; 3; 5; 7; 9. The second digit can be any one of the three digits 2; 5; 8. The third digit can be either 3 or 7. The fourth digit can be either 4 or 9. Thus we have $5 \times 3 \times 2 \times 2 = 60$ possibilities.

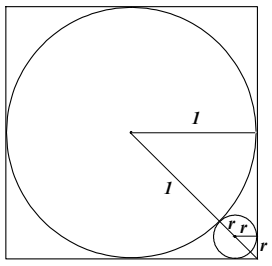
13. B. The area of Δ BMP is $\frac{1}{2}$.MN.BP, while MN = $\frac{1}{2}$ AB = $\frac{1}{2}$. The area of the square is 1, so we have been given that the area of Δ BMP is $\frac{1}{5}$. Thus $\frac{1}{2}.\frac{1}{2}.BP=\frac{1}{5}$, and hence BP = $\frac{4}{5}$



14. C. By symmetry DP = x so PC = $3 - x$. The required area is area Δ ADC − area Δ POC, which is
$$\frac{1}{2}.3.3-\frac{1}{2}(3-x).\frac{3}{2}=\frac{3}{4}(6-3+x)=\frac{3}{4}(x+3)$$



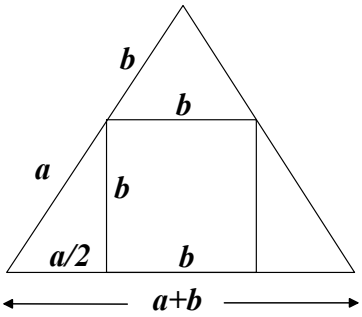
15. B. If the radius of the small circle is r, then we see that half the diagonal of the square is $1+r+\sqrt{r^2+r^2}$, while it is also $\sqrt{1^2+1^2}$.
We thus have $1+r+r\sqrt{2}=\sqrt{2}$, or $r\left(1+\sqrt{2}\right)=\sqrt{2}-1$



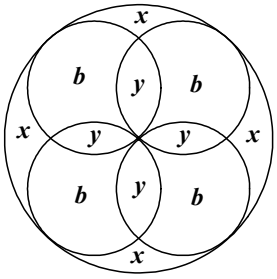
Part C: (Each correct answer is worth 6 marks)

16. A. We start with 900 cm^3 water and 100 cm^3 oil. At the end of the day we have 450 cm^3 water and 100 cm^3 oil, so the proportion of water is
$\frac{450}{450+100}=\frac{450}{550}=\frac{9}{11}=\frac{81}{99}$ which is a bit over $\frac{81}{100}$ or 81%, so 82% must be the correct response.

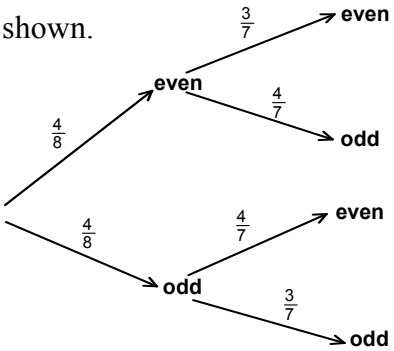
17. D. Other lengths must be as shown in the diagram; but then Pythagoras' Theorem gives $a^2 = \frac{1}{4}a^2 + b^2$, which means that $b^2 = \frac{3}{4}a^2$ and therefore $\frac{a}{b} = \frac{2}{\sqrt{3}}$



18. E. With areas as labeled, the large circle has area $4(b + y + x)$ and a small circle has area $b + 2y$. But the large circle has twice the radius of the smaller one, so four times the area. Thus $4(b + y + x) = 4(b + 2y)$ so that $b + y + x = b + 2y$ and then $x = y$, so the shaded areas are equal.



19. B. A tree diagram for the two cards drawn is as shown. The only way that the sum from the two cards can be even is if both are even or both are odd, so the probability is $\frac{4}{8}.\frac{3}{7} + \frac{4}{8}.\frac{3}{7} = 2.\frac{1}{2}.\frac{3}{7} = \frac{3}{7}$



20. C. If Adam plays soccer or cricket then by (3) Chris does not play rugby. But then (4) shows Bob does not play rugby, which would mean that no-one plays rugby even though all boys play different sports. So Adam cannot play soccer or cricket, and therefore must play rugby. Now (1) shows that Bob does not play soccer, so he must play cricket and then Chris plays soccer. Thus statement C is the only correct one.