SAMO 2009 – Junior Second Round SOLUTIONS

Part A: (Each correct answer is worth 4 marks)

1. A. If x is the number he thought of, he ends up with $(x - 3)^2 + 1 = 10$, so $(x - 3)^2$ is 9, and then since x is positive, $x - 3 = 3$ and therefore $x = 6$.

2. C. $\frac{x}{y} = \frac{3}{4}$ and $\frac{y}{z} = \frac{3}{5}$ and multiplying gives $\frac{x}{z} = \frac{9}{20}$

3. C. Estimating the square root of 2009 reveals that 2009 is near to $45^2 = 2025$ – which would need 16 to be added to 2009.

4. B. Since $3 \times C$ ends with C, where C is a digit, C must be 5. But then $3 \times ABC$ is 555, so ABC is 185, and then $A + B + C = 14$.

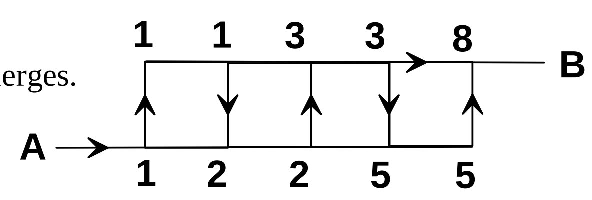
5. D. We need $5b = 5 + b$, so that $4b = 5$ and then $b = \frac{5}{4}$

Part B: (Each correct answer is worth 5 marks)

6. D. Let the length be L and the breadth be B; then $LB = 2L + 2B$. Therefore $B = \frac{2L}{L-2}$ with L and B both integers; trial shows that the only possibilities are L = 3 and B = 6, or L = 6 and B = 3, but we know L > B.

7. B. The number of terms in any row is the same as the number of the row. The total number of terms in the first 20 rows is $1 + 2 + 3 + \ldots + 20 = 210$, and so the number at the end of the 20^{th} row is 211 (the first row begins with 2)
OR recognise that the last numbers in the rows are the triangular numbers plus 1, so the last number in the 20^{th} row is $\frac{1}{2}.20.21 + 1 = 211$

8. B. At each vertex we write the number of ways of reaching it: a clear pattern emerges.

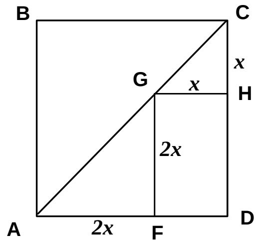


9. B. At 1 p.m. Alan has been travelling for 4 hours and has covered 80 km. Beatrice has been travelling for $3\frac{1}{2}$ hours and so has covered $3\frac{1}{2} \times 18 = 63$ km. Thus Alan is ahead by $80 - 63 = 17$ km.
OR When Beatrice starts, Alan is 10 km ahead; in each following hour Alan goes further ahead of her by 2 km, so after 3.5 hours he is $10 + 3.5(2) = 17$ km ahead.

10. B. $100! - 98! = 100 \times 99 \times 98! - 98! = 98!(100 \times 99 - 1) = (9899).98!$

11. B. TU is one-third of AC, and by Pythagoras the length of AC is $\sqrt{6^2+2^2} = \sqrt{40.} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$, so TU has length $2\frac{\sqrt{10}}{3}$

12. A. Let GH = x, so GF = $2x$. Then area of ABCD = $(3x)^2$ = $9x^2$, while area of CGH is $\frac{1}{2}x^2$. The required ratio is then $\frac{\frac{1}{2}x^2}{9x^2} = \frac{1}{18}$



13. E. $7^1 = \underline{07}$; $7^2 = \underline{49}$; $7^3 = 3\underline{43}$; $7^4 = 24\underline{01}$; $7^5 = \ldots\underline{07}$; $7^6 = \ldots\underline{49}$. The pattern for the last two digits repeats in cycles of 4. Since 2009 = 4 × 501 + 1, the last two digits of 7^{2009} will be the first group in the repeating cycle, i.e. 07

14. B. A is a difference of squares and therefore factorises, with neither factor being 1. D is the sum of two even numbers, so it must be even; E is the sum of two odd numbers and must be even. The last digit of C is the sum of the last digit of 99^2 and 98^2, which means it is 1 + 4 = 5, hence it is divisible by 5 and therefore not prime. B does factorise, because it is a difference of squares, but the factorisation is 1 × 197, and 197 is prime.

15. D. Possible arrangements are: 20098 with a score of 2 + 0 + 9 + 1 = 12; 20908 with a score of 2 + 9 + 9 + 8 = 28, 28090 with a score of 6 + 8 + 9 + 9 = 32 etc. The largest score is achieved by having the 9 and 8 adjacent to zeroes as far as possible and not at the start or end of the number.

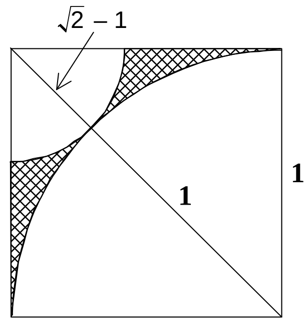
Part C: (Each correct answer is worth 6 marks)

16. E. He plays more than once. If he plays twice he can score 24, 25 or 26. If he plays three times he can score 36, 37, 38 or 39.....if he plays 7 times he can score anything from 84 to 91 (8 possibilities). And if he plays 8 times he can score 96, 97, 98, 99 or more (these other scores being irrelevant).
This makes (3+4+...+8)+4 = 33+4 = 37 scores.

17. A. The diagonal of the square has length $\sqrt{2}$, so the radii of the arcs must be 1 and $\sqrt{2}-1$. The two quarter circles therefore have total area

$$\frac{1}{4}\pi.1^2 + \frac{1}{4}\pi\left(\sqrt{2}-1\right)^2 = \frac{1}{4}\pi\left(1+2-2\sqrt{2}+1\right)$$

$$= \frac{1}{4}\pi(4-2\sqrt{2}) = \frac{1}{2}\pi(2-\sqrt{2})$$

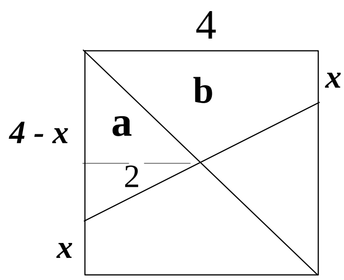


Since the area of the whole square is 1, the shaded area is $1-\frac{1}{2}\pi(2-\sqrt{2})$

18. C. By symmetry, $a + b$ is half the square, with a being one third of that half and b two-thirds of it. The area a is a triangle with base $4 - x$ and height 2.

So $\frac{1}{2}(4-x).2 = a = \frac{1}{3}.\frac{1}{2}.4^2$, giving

$4 - x = \frac{8}{3}$ and hence $x = \frac{4}{3}$



19. D. Consider the three-digit number whose digits are xyz and whose value is thus $100x + 10y + z$. When the digits are reversed the result has value $100z + 10y + x$. Subtracting (and clearly we must have $z > x$) gives $99z - 99x$, so $99(z - x) = 297$ and then $z - x = 3$. The possibilities are $x = 1$ and $z = 4$, or $x = 2$ and $z = 5$, and so on up to $x = 6$ and $z = 9$ (neither x nor z can be 0). For each choice of x and z we have a free choice for y, giving 10 possibilities. Therefore the total number of possibilities is $6 \times 10 = 60$.

20. E. The first numbers in each group form a sequence which goes up by 12 each time; that is true also for the second numbers, the third ones and the fourth ones. So the total from one group to the next increases by $12 \times 4 = 48$, and going from the first group to the sixth group will lead to an increase in total of $48 \times 5 = 240$.