

**HARMONY SOUTH AFRICAN  
MATHEMATICS OLYMPIAD**

**SECOND ROUND 2007  
JUNIOR SECTION: GRADES 8 AND 9**

**SOLUTIONS AND MODEL ANSWERS**

NUMBER	POSITION
1	B
2	B
3	A
4	D
5	E
6	B
7	A
8	C
9	D
10	C
11	A
12	B
13	A
14	E
15	E
16	D
17	B
18	C
19	E
20	D

1. **B**

$$18 = 2 \times 3^2$$

$$30 = 2 \times 3 \times 5$$

$$45 = 3^2 \times 5$$

$$\text{LCM}(18, 30, 45) = 2 \times 3^2 \times 5 = 90$$

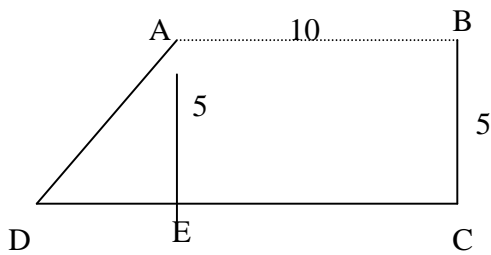
2. **B**

Subtracting multiples of 7 from 29 we obtain  $29 - 28 = 1$  (first Thursday)

First Saturday is therefore  $1 + 2 = 3^{\text{rd}}$

Second Saturday is  $3 + 7 = 10^{\text{th}}$

3. **A**



Construct  $AE \perp DC$ . Fill in the angles

$\therefore \triangle ADE$  is isosceles with  $AE = DE$

But  $AE = BC = 5$  (rectangle AECB)

$\therefore DE = 5$

$EC = AB = 10$

$\therefore y = 5 + 10 = 15$

4. **D**

Sipho's age is a multiple of 3,

e.g. 18; 21; 24; 27; 30; 33; 36; .....etc

Pam's age is the sum of the digits of Sipho's age,

e.g.  $1 + 8 = 9$ ,  $2 + 1 = 3$ ,  $2 + 4 = 6$ ,  $2 + 7 = 9$ ,  $3 + 0 = 3$ ,  $3 + 3 = 6$ , etc

Sum of their ages is :

Either 48 or 56 or 32 or 36 or 60.

Solution is therefore  $27 + (2 + 7) = 36$

5. **E**

$$\begin{aligned}x &= \frac{1}{2}(90^\circ - x) + 30^\circ \\ &= 45^\circ - \frac{x}{2} + 30^\circ\end{aligned}$$

$$\frac{3x}{2} = 75^\circ$$

$$3x = 150^\circ$$

$$x = 50^\circ$$

supplement of  $x$  is  $180^\circ - 50^\circ = 130^\circ$ .

6. **B**

$$\sqrt{xy} = 2 \Rightarrow xy = 4$$

$$\text{and } \sqrt[3]{xyz} = 6 \Rightarrow xyz = 216$$

$$z = \frac{xyz}{xy}$$

$$= \frac{216}{4} = 54$$

7. **A**

Write out the 2-digit number as  $10x+y$ , such that:

$$10x+y-(x+y) = 45$$

$$x = 5$$

Since  $y$  is independent of  $x$  it can take on any value from 0 to 9.

There are therefore 10 possible numbers

8. **C**

Say there are  $a$  five cent coins,  $b$  ten cent coins and  $c$  twenty cent coins. Then:

$$5a + 10b + 20c = 275 \dots\dots\dots(1)$$

$$10a + 20b + 5c = 375 \dots\dots\dots(2)$$

Double each one in (1)

$$10a + 20b + 40c = 550$$

$$10a + 20b + 5c = 375$$

$$35c = 175$$

$$c = 5$$

9. **D**

$$\begin{aligned}6^2 + 8^2 &= 10^2 \\8^2 + 15^2 &= 17^2 \\10^2 + 24^2 &= 26^2 \\12^2 + 35^2 &= 37^2 \\&\dots \\&\dots \\18^2 + n^2 &= x^2\end{aligned}$$

Observe pattern:  $8 + 7 = 15$ ,  $15 + 9 = 24$ ,  $24 + 11 = 35$ ,  $35 + 13 = 48$ ,  $48 + 15 = 63$ ,  
 $63 + 17 = 80$

10. **C**

Suppose there were 7 guests (1 on each day)

	Son	Mon	Tue	Wed	Thurs	Fri	Sat
	1	2	3	4	5	6	7
Suppose there were 15	8	9	10	11	12	13	14
	15						

This gives us at least 3 on one day.

For 6 months

We have  $14 \times 6 + 1 = 84 + 1$   
 $= 85$

11. **A**

Consider:  $x = 1$ (no.)

$$a + b + c + d = 15$$

Combinations: (2;2;5;6)

(2;3;4;6)

(2;3;5;5)

(3;3;3;6)

(3;3;4;5)

(4;4;4;3)

(4;4;2;5)

Consider:  $x = 2$

$$a + b + c + d = 10$$

(1;1;3;5)

(1;1;4;4)

(3;3;3;1)

Possible number of combinations is 10

12.

**B**

$$2 + 4 = 6 = 2 \times 3 \quad \dots\dots \text{No. of terms} \times \text{the next number of terms.}$$

$$2 + 4 + 6 = 12 = 3 \times 4$$

$$2 + 4 + 6 + 8 = 20 = 4 \times 5$$

$$2 + 4 + 6 + 8 + 10 = 30 = 5 \times 6$$

$$\begin{aligned} \text{Therefore } 2 + 4 + 6 + 8 + 10 + \dots + 30 &= 15 \times 16 \\ &= 240 \end{aligned}$$

13.

**A**

Consider the quadrilateral with  $AB = BC = CD$

$$\hat{C} = 60^\circ \quad \hat{B} = 100^\circ$$

Join BD

Now triangle BDC is equilateral.

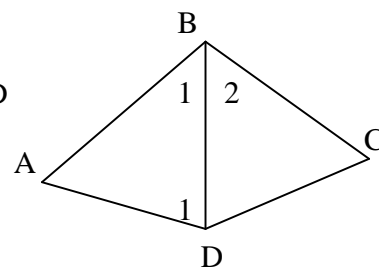
$$\text{So } \hat{B}_2 = 60^\circ$$

$$\therefore \hat{B}_1 = 40^\circ$$

And  $BA = BD$

$$\text{So } \hat{A} = \hat{D}_1 = 70^\circ \quad (\text{angle sum of isos triangle})$$

$$\therefore \hat{D} = 70^\circ + 60^\circ = 130^\circ$$



14.

**E**

$$a = 15 + 5 = 20$$

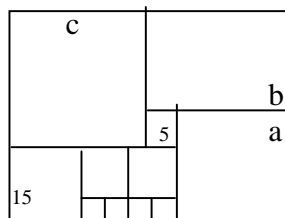
$$b = 20 + 5 = 25$$

$$c = 25 + 5 = 30$$

So length is  $30 + 25 = 55$  and

breadth is  $25 + 20 = 45$ .

$$\text{Area is } 45 \times 55 = 2475$$



15. **E**

$$\text{Water + melon} = 50\text{kg.}$$

$$\begin{aligned}\text{Mass of water} &= 98\% \text{ of } 50 \\ &= 49 \text{ kg.}\end{aligned}$$

$$\text{Mass of the solid matter in the melon} = 1 \text{ kg.}$$

4% of watermelon represents 1 kg.

$$\begin{aligned}\text{Therefore new mass of watermelon} &= \frac{100}{4} \times 1 \text{ kg} \\ &= 25\text{kg}\end{aligned}$$

16. **D**

$$\text{Laat: } 478 = a$$

$$392 = b$$

$$\text{and } 263 = c$$

$$a - b = 86 = 2 \times 43$$

$$b - c = 129 = 3 \times 43$$

$$\text{Therefore } 263 = 6 \times 43 + 5$$

$$N = 43$$

$$P = 5$$

$$N - P = 43 - 5$$

$$= 38$$

17. **B**

$$\begin{aligned}3 + \sqrt{27} &= 3 + 3\sqrt{3} = 3 + \sqrt{3} + \sqrt{3} + \sqrt{3} \\ &< 3 + \sqrt{4} + \sqrt{3} + \sqrt{3} = 5 + \sqrt{12}\end{aligned}$$

$$\begin{aligned}\sqrt{48} + \sqrt{3} &= \sqrt{16 \times 3} + \sqrt{3} = 4\sqrt{3} + \sqrt{3} \\ &= 3\sqrt{3} + \sqrt{3} + \sqrt{3} = \sqrt{27} + 2\sqrt{3} = \sqrt{27} + \sqrt{12} \\ &> \sqrt{27} + \sqrt{9} = 3 + \sqrt{27}\end{aligned}$$

also

$$\sqrt{48} + \sqrt{3} = \sqrt{27} + \sqrt{12} > \sqrt{25} + \sqrt{12} = 5 + \sqrt{12}$$

$$\sqrt{78} > \sqrt{75} = 5\sqrt{3} = 4\sqrt{3} + \sqrt{3} = \sqrt{48} + \sqrt{3}$$

$$3 + \sqrt{3} + \sqrt{10} < 3 + \sqrt{3} + \sqrt{12} = 3 + \sqrt{27}$$

so

$$3 + \sqrt{3} + \sqrt{10} < 3 + \sqrt{27} < 5 + \sqrt{12} < \sqrt{48} + \sqrt{3} < \sqrt{78}$$

$5 + \sqrt{12}$  is the number in the middle

18. C

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	....
1	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	o	
2		x		x		x		x		x		x		x		x		
3			x			o			x			o			x			
4				o				o				x				o		
5					x					o					o			
6						x						o						
7							x							o				
8								x								x		
9									o									
10										x	x	x	x	x	x	o	x	

$$\begin{aligned} \text{Number off} &= 80 - 8 \\ &= 72 \end{aligned}$$

72 lights will be switched off

19. E

Jessie walked  $8 \times 80 = 640$  m in 10 min so her speed was 64 m per min  
 With conveyor system she walked 320m at 64m per min which took her  $320/64 = 5$  min, she then took  $8 - 5 = 3$  min on the conveyor system,  
 The conveyor moves 320m in 3 min which is at a speed of  $320/3 = 106 \frac{2}{3}$  m per min. Walking on the conveyor system she takes 5 min plus  $320/(106 \frac{2}{3} + 64)$   
 This is approximately 7 min.

20. D

The sum of the digits in the answer is: when 9 rows are added  $9 \times 4 + 9 = 45$

$$\begin{aligned} \text{when 18 rows are added } & 18 \times 4 + 9 = 81 \\ & 27 \qquad \qquad \qquad 27 \times 4 + 9 = 117 \\ & \dots\dots\dots \\ & \dots\dots\dots \\ & \dots\dots\dots \\ & 63 \qquad \qquad \qquad 63 \times 4 + 9 = 261 \end{aligned}$$