

**THE HARMONY SOUTH AFRICAN
MATHEMATICS OLYMPIAD**

**SECOND ROUND 2006
JUNIOR SECTION: GRADES 8 AND 9**

SOLUTIONS AND MODEL ANSWERS

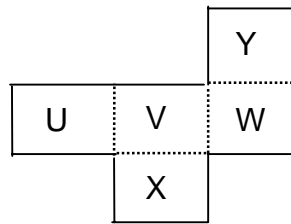
NUMBER	POSITION
1	E
2	B
3	A
4	C
5	E
6	B
7	D
8	D
9	C
10	C
11	E
12	E
13	A
14	C
15	B
16	A
17	C
18	D
19	C
20	E

PART A

1. $15\% \times R560 - 15\% \times R500$
 $=15\% \times (R560-R500)$
 $=15\% \times R60$
 $=R9$

Answer : E

2.



Fold U, W, X down and Y around such that:

- X and W share a common edge
- X and U share a common edge
- U and Y share a common edge
- Y and W share a common edge
- V shares a common edge with U, X, W and Y

Therefore, V is the label at the bottom.

Answer: B

3. Raising each of the five numbers to the power of 6 gives:

$$1^6 = 1$$

$$\left(\sqrt[3]{9}\right)^6 = 9^2 = 81$$

$$2^6 = 64$$

$$\left(\sqrt{5}\right)^6 = 5^3 = 125$$

$$3^6 = 729$$

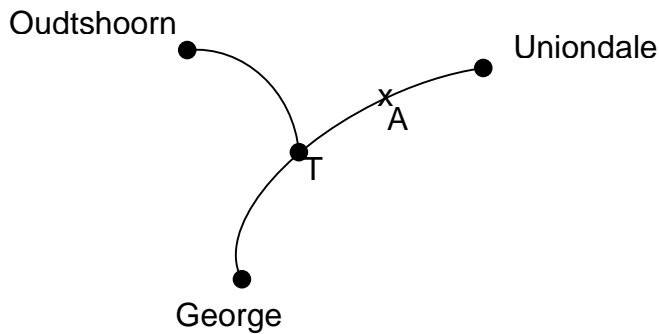
The order in terms of magnitude is:

1; 64; 81; 125; 729 or

1; 2; $\sqrt[3]{9}$; $\sqrt{5}$; 3

Answer : A

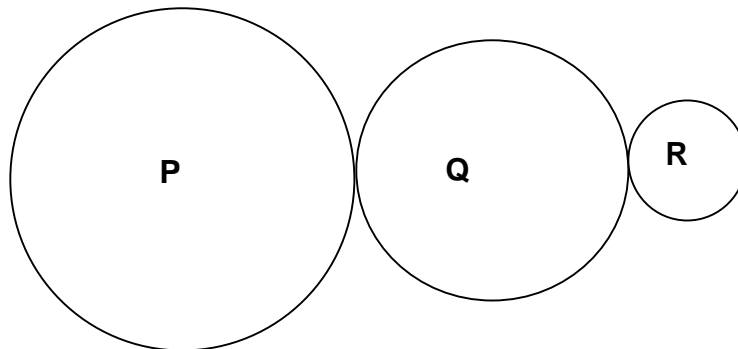
4.



Distance from Oudtshoorn to George is $(68-34) + (60-34) = 34+26 = 60\text{km}$.

Answer : C

5.



When wheel P starts to turn, the adjacent wheel Q will turn in the opposite direction. After one revolution of 360 degrees a point on the circumference of P will have moved through $2(\pi) \cdot 36 = 72(\pi)$ cm. A point on the circumference of Q will move through the same distance. But the circumference of Q is $2(\pi) \cdot 12 = 24(\pi)$ cm which goes into $72(\pi)$ three times. Therefore Q makes three anti-clockwise revolutions for every one revolution of P. Similarly every revolution of Q makes R go through 2 revolutions clockwise. Therefore the answer is $3 \times 2 = 6$ revolutions of 360 degrees each.

Answer: E

PART B

6.

L	G	D	O
D	O	L	G
O	L	G	D
G	D	O	L

Completed grid using logic is shown above. Required letter is in the third row and first column.

Answer: B

7. Ratio of humans to ants = $\frac{1 \times 10^{16}}{6 \times 10^9} = 1$ to $1,67 \times 10^6$

Answer: D

8. Distance = 15km; time = 11/60 hour; speed = x

$$speed = \frac{distance}{time}$$

$$\begin{aligned} x &= 15 \div \frac{11}{60} \\ &= \frac{15 \times 60}{11} \\ &= \frac{900}{11} \quad (= 81, 82) \end{aligned}$$

$$\therefore 80 \leq x < 90$$

Answer: D

9. $5 * x = \frac{5x}{x+5} = 2 \text{ rem } 5$

$$5x = 2(x+5) + 5$$

$$x = 5$$

Answer: C

10. 1; 3; 7; 15;....

Pattern is powers of 2 minus 1, i.e. $2^n - 1$

When $n = 15$, we get $2^{15} - 1$

Answer: C

11. If n is a perfect square then \sqrt{n} is an integer.

Therefore the next perfect square is $(\sqrt{n} + 1)^2 = n + 2\sqrt{n} + 1$

Answer: E

12. $\frac{3}{7} = 0,428571\ 428571\ 428571\ \dots$

so every sixth digit is a "1"

But 2004 is a multiple of 6 so a_{2004} is also 1.

Also, when 2006 is divided by 6, the remainder is 2, so counting the second digit in the repeated block 428571 will give $a_{2006} = 2$.

Answer: E

13. Consider $4m61n2$

$$\text{Sum of alternate digits} = 4+6+n = m+1+2$$

$$10+n = m+3$$

For divisibility by 4, the digit n has to be odd.

For divisibility by 4 and 11:

$$n=1 \Rightarrow m+3 = 11 \Rightarrow m=8$$

$$n=3 \Rightarrow m+3 = 13 \Rightarrow m=10, \text{ which is not possible, } m \leq 9$$

$$n=5 \Rightarrow m+3 = 15 \Rightarrow m=12, \text{ which is not possible, } m \leq 9$$

$$\text{or } 15 - m - 3 = k \times 11$$

$$\Rightarrow m = 1$$

$$n=7 \Rightarrow m+3 = 17 \Rightarrow m=14, \text{ which is not possible, } m \leq 9$$

$$\text{or } 17 - m - 3 = k \times 11$$

$$\Rightarrow m=3$$

$$n=9 \Rightarrow m+3 = 19 \Rightarrow m=16, \text{ which is not possible, } m \leq 9$$

$$\text{or } 19 - m - 3 = k \times 11$$

$$\Rightarrow m=5$$

Answer: A

14.

Thousands	Hundreds	Tens	Units
	4	x	$x+3$
	4	$x+3$	x
	x	4	$x+3$
	x	$x+3$	4
	$x+3$	x	4
	$x+3$	4	x
2	8	8	6

The sum of the numbers in the units column is $4x + 14$

Consider:

$$4x+14 = 6 \quad (X)$$

$$4x+14 = 16$$

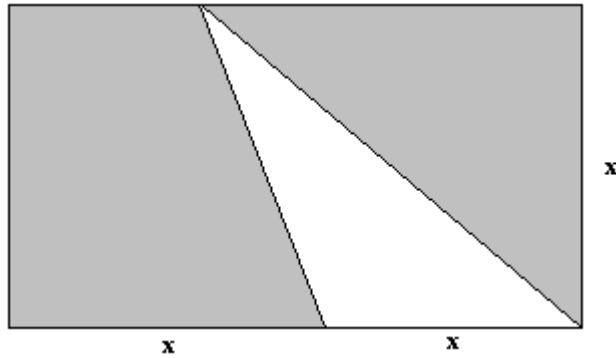
$$x = 1/2 \quad (X)$$

$$4x + 14 = 26$$

$x = 3$ and $x+3 = 6$ such that the three digits are 3; 6 and 4

Answer: C

15.



$$\text{Total Area of rectangle} = 2x(x) = 2x^2$$

$$\text{Area of triangle} = \frac{1}{2} \cdot \text{Base} \times \text{height}$$

$$\text{Area of unshaded triangle} = \frac{1}{2} \times x \times x = \frac{x^2}{2}$$

$$\text{Area of shaded region} = \frac{3x^2}{2}$$

Answer: B

PART C

16. Sum of digits is 187, which is not divisible by 3. Therefore $k=0$.

Answer: A

17. $14 \times 14 = 232$

$$14^2 = 232$$

$$[x + 4x^0]^2 = 2x^2 + 3x + 2$$

$$x^2 + 8x + 16 = 2x^2 + 3x + 2$$

$$x^2 - 5x - 14 = 0$$

$$(x - 7)(x + 2) = 0$$

$$x = 7 \text{ or } x = -2$$

Answer: C

18. $4(x + y + z) = 68$ so that $x + y + z = 17$ and $y \times z = 18$
 $\therefore y$ and z can either be 2 and 9 or 3 and 6
 $\therefore x = 6$ or 8
 \therefore possible volumes are $2 \times 9 \times 6 = 108$ or $3 \times 6 \times 8 = 144$

Answer: D

19. A corner cube shows three faces. The other two cubes along each of the edges of the cube show 2 faces. The 4 cubes in the middle of each face show only one face. We can hide 8 of the black cubes right in the middle of the large cube. They will show no black faces. Then we can put 4 black cubes in the middle of each of the 6 faces. That means that we have used up $8 + 4 \times 6 = 32$ black cubes. So we have one more black cube left. This last one can be put along one of the edges and shows 2 black faces. Therefore the least number of black squares is $4 \times 6 + 2 = 26$.

Answer: C

20. To form a triangle we need to choose 3 out of the 9 points as vertices. Note, if we choose three collinear points then we don't have a triangle. The first vertex can be chosen in 9 ways. The second in 8 ways and the third in 7 ways. That makes a total of $9 \times 8 \times 7$ ways. But that leads to lots of repetitions of the same triangle, because the order in which we choose the vertices does not matter. Once we have chosen three vertices, they can be listed in $3 \times 2 \times 1 = 6$ different ways. So in the $9 \times 8 \times 7$ ways of choosing a triangle every triangle is repeated 6 times. Therefore we have to divide $9 \times 8 \times 7$ by 6. This comes to 84. But there are 8 "flat" triangles formed by collinear points (each row, each column and every diagonal of three points). Therefore the answer is $84 - 8 = 76$.

Answer: E