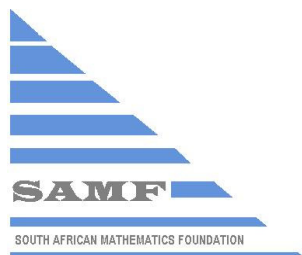




SOUTH AFRICAN MATHEMATICS OLYMPIAD



Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2010 SECOND ROUND JUNIOR SECTION: GRADES 8 AND 9

18 May 2010

Time: 120 minutes

Number of questions: 20

Instructions

1. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
2. Scoring rules:
 - 2.1. Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.
 - 2.2. For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
3. You must use an HB pencil. Rough paper, a ruler and an eraser are permitted. **Calculators and geometry instruments are not permitted.**
4. Diagrams are not necessarily drawn to scale.
5. Indicate your answers on the sheet provided.
6. Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.
7. Answers and solutions will be available at www.samf.ac.za

***Do not turn the page until you are told to do so
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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns



PRACTICE EXAMPLES

1. $23 + 6 - 4 =$

- (A) 6 (B) 23 (C) 25 (D) 29 (E) 33

2. $\frac{1}{5} + \frac{2}{3} \times \frac{1}{2}$ equals

- (A) $\frac{1}{15}$ (B) $\frac{3}{11}$ (C) $\frac{21}{50}$ (D) $\frac{8}{15}$ (E) $9\frac{4}{5}$

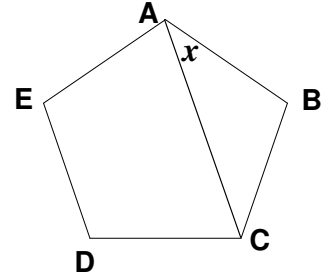
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Part A: (Each correct answer is worth 4 marks)

1. The number 2010 has the property that the number formed by its first two digits is twice the number formed by its last two digits. The number of four-digit numbers with this property is

(A) 9 (B) 10 (C) 40 (D) 45 (E) 50

2. ABCDE is a regular pentagon. The size of the angle marked x is



(A) 18° (B) 27° (C) 33° (D) 36° (E) 42°

3. A rectangular tank has dimensions 2 m by 2 m by 4 m. Water fills the tank at a rate of 5 litres per second. The number of minutes required to fill the tank is nearest to

(A) 3 (B) 50 (C) 53 (D) 55 (E) 60

4. Today John has X CDs. Tomorrow he will give Jane four of them, and then she will have twice as many as he will have. The number of CDs that Jane has today is

(A) $2X - 4$ (B) $2(X - 4)$ (C) $2X + 4$ (D) $2(X - 6)$ (E) $2X + 8$

5. If $a + b = c - d$ and $a + c = b - d$, then $a + d$ is

(A) b (B) c (C) $-c$ (D) 0 (E) $-b$

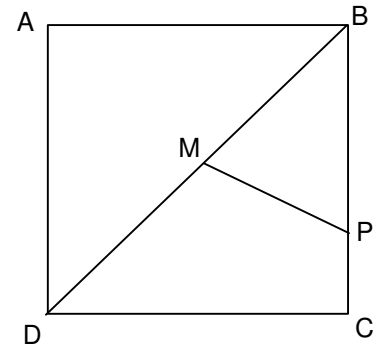
Part B: (Each correct answer is worth 5 marks)

6. Charlie is now twice as old as Arthur, and four years older than Bernard. In six years' time the sum of all their ages will be 69. Bernard's age now is

(A) 11 (B) 13 (C) 18 (D) 22 (E) 26

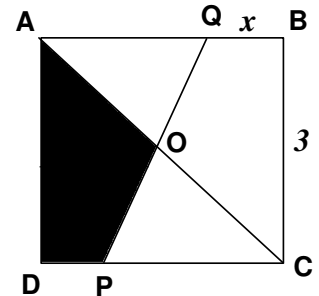
7. Some of the natural numbers less than 1000 leave a remainder of 1 when divided by 21. The number of these that also leave a remainder of 1 when divided by 35 is
- (A) 8 (B) 9 (C) 10 (D) 11 (E) 12
8. Among the pupils at a school, the ratio of seniors to juniors is 5 : 3. Among the juniors the ratio of boys to girls is 3 : 2, and among the seniors it is 2 : 3. The ratio of boys to girls in the whole school is
- (A) 19 : 21 (B) 17 : 21 (C) 15 : 19 (D) 5 : 3 (E) 5 : 4
9. John cycles 10 km/h faster than Dave, and takes one third of the time that Dave takes. They both cover the same distance. Dave's speed in km/h is
- (A) 4 (B) 5 (C) 6 (D) 7 (E) 8
10. If $4^x \times 6^y = 48^{12}$ then the value of $x + y$ is
- (A) 24 (B) 30 (C) 48 (D) 36 (E) 32
11. The value of $1 - 4 + 9 - 16 + 25 + \dots + 625$ is
- (A) 725 (B) 575 (C) 425 (D) 375 (E) 325
12. Lebo is asked to find a four-digit number such that the first digit leaves a remainder of 1 when divided by 2, the second digit leaves a remainder of 2 when divided by 3, the third digit leaves a remainder of 3 when divided by 4 and the last digit leaves a remainder of 4 when divided by 5. The number of different four-digit numbers he can find with this property is
- (A) 48 (B) 60 (C) 80 (D) 120 (E) 180

13. The midpoint M of a square $ABCD$ is joined to a point P on the side BC . If the square has sides of length 1 unit, and the area of $\triangle BMP$ is $\frac{1}{5}$ that of the square, then the length BP is



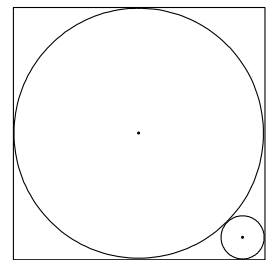
- (A) $\frac{3}{5}$ (B) $\frac{4}{5}$ (C) $\frac{5}{9}$ (D) $\frac{7}{9}$ (E) $\frac{7}{10}$

14. $ABCD$ is a square of side 3 units. PQ is a line through O , the centre of the square, meeting AB at a distance x units from B . The shaded area in terms of x is



- (A) $\frac{x+9}{4}$ (B) $\frac{2x+3}{2}$ (C) $\frac{3}{4}(x+3)$ (D) $\frac{9}{4}(5-x)$ (E) $\frac{9}{16}(4-x)$

15. The figure shows a square with a circle just fitting into it and another circle fitting into the corner. The sides of the square are of length 2 units. The radius of the smaller circle is



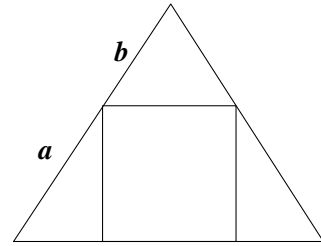
- (A) $\frac{\sqrt{2}-1}{2}$ (B) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (C) $\sqrt{2}-1$ (D) $\frac{1}{2}$ (E) $\frac{\sqrt{2}}{1+2\sqrt{2}}$

Part C: (Each correct answer is worth 6 marks)

16. At the beginning of the day I have $1\,000\text{ cm}^3$ of a mixture that is 90% water and 10% oil. Evaporation during the day means that we lose 50% of the water and none of the oil. The percentage of water in the remaining mixture is approximately

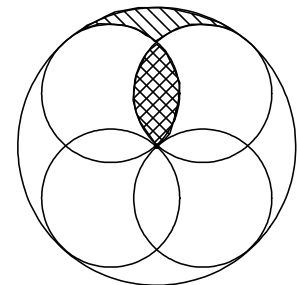
(A) 82% (B) 78% (C) 50% (D) 45% (E) 35%

17. The figure shows an equilateral triangle with a square fitting inside it: all four vertices of the square lie on sides of the triangle. The value of $\frac{a}{b}$ is



(A) $\frac{3}{2}(\sqrt{3}-1)$ (B) $\frac{\sqrt{3}+1}{2}$ (C) $\frac{1}{4}(4+\sqrt{3})$ (D) $\frac{2}{\sqrt{3}}$ (E) $\sqrt{3}$

18. Four circles of equal size lie in a circle so that each touches the larger circle and also passes through its centre. The arrangement is symmetrical, with the centres of the smaller circles all on the vertices of a square. The larger circle has radius 2 units. The difference in area between the two shaded parts is



(A) $\frac{1}{8}$ (B) $\frac{\pi}{12}$ (C) $\frac{\pi}{18}$ (D) $\frac{\pi}{24}$ (E) 0

19. 8 cards all show different numbers; four of those numbers are even, the others are odd. If two of the cards are chosen at random, the probability that the sum of their numbers is even is

(A) $\frac{3}{8}$ (B) $\frac{3}{7}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{5}{7}$

20. Adam, Bob and Chris play different sports. Four statements are true:

- (1) If Bob plays soccer, Adam plays cricket.
- (2) If Bob plays rugby, Adam plays soccer.
- (3) If Adam plays soccer or cricket, Chris does not play rugby.
- (4) If Chris does not play rugby, Bob does not play rugby.

Which one of the following statements must be correct?

- (A) Bob plays soccer
- (B) Bob plays rugby
- (C) Adam plays rugby
- (D) Chris plays cricket
- (E) Adam plays soccer