SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the
SOUTH AFRICAN MATHEMATICS FOUNDATION

2009 SECOND ROUND
JUNIOR SECTION: GRADES 8 AND 9

21 May 2009 **Time: 120 minutes** **Number of questions: 20**

Instructions

1. *Do not open this booklet until told to do so by the invigilator.*
2. *This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.*
3. *Scoring rules:*
   3.1. *Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.*
   3.2. *For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.*
4. *You must use an HB pencil. Rough paper, a ruler and an eraser are permitted.* ***Calculators and geometry instruments are not permitted.***
5. *Diagrams are not necessarily drawn to scale.*
6. *Indicate your answers on the sheet provided.*
7. *Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.*
8. *Answers and solutions will be available at* `www.samf.ac.za`

Do not turn the page until you are told to do so
Draai die boekie om vir die Afrikaanse vraestel

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Organisations involved: AMESA, SA Mathematical Society,
SA Akademie vir Wetenskap en Kuns

PRACTICE EXAMPLES

1. $23 + 6 - 4 =$

(A) 6 (B) 23 (C) 25 (D) 29 (E) 33

2. $\frac{1}{5}+\frac{2}{3}\times\frac{1}{2}$ equals

(A) $\frac{1}{15}$ (B) $\frac{3}{11}$ (C) $\frac{21}{50}$ (D) $\frac{8}{15}$ (E) $9\frac{4}{5}$

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Part A: (Each correct answer is worth 4 marks)

1. Andrew thinks of a natural number, reduces it by 3, squares the result and then adds 1. If he ends up with 10, the number he thought of was:

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

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2. If the ratio $x : y$ is 3 : 4 and the ratio $y : z$ is 3 : 5, then the ratio $x : z$ is:

(A) 4 : 5 (B) 5 : 4 (C) 9 : 20 (D) 20 : 9 (E) 4 : 9

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3. The smallest positive integer which must be added to 2009 in order to get a perfect square is:

(A) 7 (B) 9 (C) 16 (D) 25 (E) 41

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4. ABC is a three-digit number such that

$$\begin{array}{rr} & A\,B\,C \\ & A\,B\,C \\ + & A\,B\,C \\ \hline & C\,C\,C \end{array}$$

The sum of the digits A, B and C is:

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

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5. If $ab = a + b$ then we say that b is the 'sumprod partner' of a. The sumprod partner of 5 is:

(A) $\frac{6}{5}$ (B) $\frac{5}{6}$ (C) $\frac{4}{5}$ (D) $\frac{5}{4}$ (E) 4

Part B: (Each correct answer is worth 5 marks)

6. A rectangle has an area which is numerically equal to its perimeter, where both the length and the breadth are integers. The rectangle is not a square.
The length of the shortest side of the rectangle is:

(A) 8 (B) 6 (C) 5 (D) 3 (E) 1

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7. The integers greater than 1 are written in a pattern as shown:

Row 1: 2
Row 2: 3 4
Row 3: 5 6 7
Row 4: 8 9 10 11
etc

What is the last number in the 20^{th} row?

(A) 210 (B) 211 (C) 212 (D) 213 (E) 214

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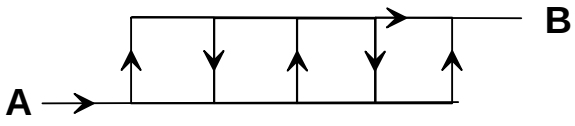
8. This map shows a grid of one-way streets.
How many different routes are there from A to B?



(A) 10 (B) 8 (C) 7 (D) 5 (E) 3

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9. Alan leaves Cape Town at 9 a.m. travelling at a constant speed of 20 km/h. Half an hour later Beatrice sets out from the same place along the same road, but at a constant speed of 18 km/h.
At 1 p.m. Alan is ahead of Beatrice by:

(A) 8 km (B) 17 km (C) 63 km (D) 80 km (E) 143 km

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10. Did you know … $2! = 2 \times 1$
$3! = 3 \times 2 \times 1$
$4! = 4 \times 3 \times 2 \times 1$?

Therefore $100! - 98!$ is equal to

(A) $9889(98!)$ (B) $9899(98!)$ (C) $9900(98!)$ (D) $9901(98!)$ (E) $9902(98!)$

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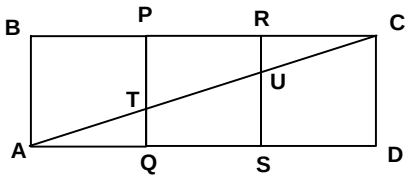
11. The figure shows a rectangle made up of three squares. If the side of each square is 2 units, and ATUC is a diagonal of the rectangle, the length of TU is:



(A) $4\frac{\sqrt{10}}{3}$ (B) $2\frac{\sqrt{10}}{3}$ (C) $\frac{\sqrt{10}}{3}$ (D) $3\frac{\sqrt{10}}{2}$ (E) $3\frac{\sqrt{10}}{4}$

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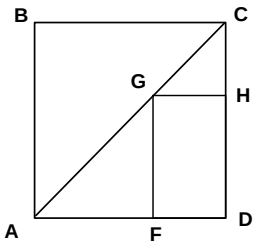
12. DFGH is a small rectangle inside a larger square ABCD, with GF = 2GH and G is on the diagonal AC.
The ratio of the areas of Δ CGH and square ABCD is:



(A) 1 : 18 (B) 1 : 9 (C) 1 : 8 (D) 2 : 9 (E) 1 : 12

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13. The last two digits of 7^{2009} are:

(A) 56 (B) 01 (C) 49 (D) 43 (E) 07

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14. Which one of the following is prime?

(A) $99^2 - 97^2$ (B) $99^2 - 98^2$ (C) $99^2 + 98^2$ (D) $98^2 + 96^2$ (E) $99^2 + 97^2$

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15. The digits of 20098 can be arranged in any order. For each arrangement, the 'score' is the sum of the positive differences between successive digits. The maximum score that can be achieved is:

(A) 24 (B) 27 (C) 28 (D) 32 (E) 34

Part C: (Each correct answer is worth 6 marks)

16. In a certain game a player can score either 12 or 13 points. Vusani plays this game more than once and adds all his scores to get a total score. The number of total scores less than 100 which are possible is:

(A) 15 (B) 25 (C) 31 (D) 37 (E) 39

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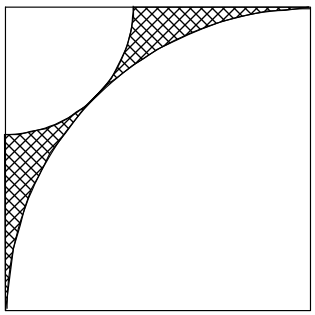
17. Quarter circles are drawn centred on opposite vertices of a square, just touching each other. One of the arcs goes through two vertices of the square. If the square has sides of length 1 unit, then the shaded area is:



(A) $1-\frac{\pi}{2}\left(2-\sqrt{2}\right)$ (B) $1-\frac{\pi}{2}\left(\sqrt{2}-2\right)$ (C) $1-\pi\left(2-\sqrt{2}\right)$

(D) $1-\frac{\pi}{2}\left(\sqrt{2}-1\right)$ (E) $\frac{\sqrt{2}\pi}{2}-1$

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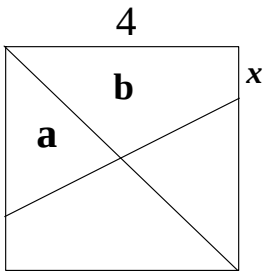
18. The diagram shows a square, with one diagonal drawn. A second line is drawn which passes through the centre of the square and meets two sides at a distance x from a corner. The square has sides of length 4 cm. The letters a and b represent the areas of the regions, and $b = 2a$. The value of x is:



(A) 8/3 (B) 7/3 (C) 4/3 (D) 5/3 (E) 2/3

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19. Some three-digit numbers have a special property. For each of these numbers, when its digits are reversed the new number is bigger than the original by 297. How many different three-digit numbers have this property?

(A) 24 (B) 36 (C) 54 (D) 60 (E) 72

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20. A pattern goes as follows:

2; 5; 8; 11;	14; 17; 20; 23;	26; 29; ….

The numbers in the pattern are added in groups of four as shown. The difference between the sum of the numbers in the sixth group and the sum of the numbers in the first group is:

(A) 266 (B) 260 (C) 254 (D) 248 (E) 240