

THE SOUTH AFRICAN MATHEMATICS OLYMPIAD



Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION

**SECOND ROUND 2008
JUNIOR SECTION: GRADES 8 AND 9**

**22 MAY 2008
TIME: 120 MINUTES
NUMBER OF QUESTIONS: 20**

Instructions

- 1) Do not open this booklet until told to do so by the invigilator.
- 2) This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
- 3) Scoring rules:
 - a) Each correct answer is worth 4 marks in part A, 5 marks in part B and 6 marks in part C.
 - b) For each incorrect answer one mark will be deducted. There is no penalty for unanswered questions.
- 4) You must use an HB pencil. Rough paper, a ruler and an eraser are permitted. **Calculators and geometry instruments are not permitted.**
- 5) Diagrams are not necessarily drawn to scale.
- 6) The centre page is an information and formula sheet. Please tear it out for your use.
- 7) Indicate your answers on the sheet provided.
- 8) Start when the invigilator tells you to do so. You have 120 minutes to complete the question paper.
- 9) Answers and solutions will be available at www.samf.ac.za in June.

**DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO
DRAAI DIE BOEKIE OM VIR DIE AFRIKAANSE VRAESTEL**

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Organisations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

Part A: Four marks each.

1. How many of the following numbers are divisible by 48?

1008; 2008; 3008; 4008; 5008

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

2. How many pairs of positive integers (m, n) satisfy $m^n = 16$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

3. Evaluate:

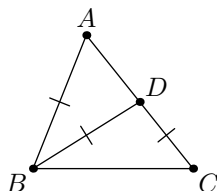
$$3 - \frac{2}{3 - \frac{2}{3}}$$

(A) $-\frac{5}{3}$ (B) $-\frac{2}{11}$ (C) $\frac{11}{6}$ (D) $\frac{15}{7}$ (E) $\frac{28}{11}$

4. Siphso buys shares in the stock market for R500. Over the next three years, the value of the shares increases by 40% and Siphso sells his shares at that price. If he pays a 7% transaction fee on both his purchase price and sale price, then how much profit does Siphso make?

(A) R116 (B) R130 (C) R151 (D) R172 (E) R193

5. In the given triangle, $AB = BD = DC$ and $\widehat{ABD} = \widehat{BCD}$. Find the size of \widehat{BAC} .



(A) 36° (B) 45° (C) 60° (D) 72° (E) 75°

Part B: Five marks each.

6. A three digit number has $(2x + 1)$ as its hundreds digit, $(x - 1)$ as its tens digit and x as its units digit. The three digit number, in terms of x , is
(A) $4x$ (B) $2x^3 - x^2 - x$ (C) $211x + 90$ (D) $211 + 90x$
(E) $211x^2 + 90x$
-

7. If $a + b = 23$ and $a^2 - b^2 = 23$, then what is the value of a ?
(A) 12 (B) 13 (C) 14 (D) 15 (E) 16
-

8. What is the smallest value of n such that the product $n! = 1 \times 2 \times 3 \times \dots \times n$ ends in at least 10 zeroes?
(A) 30 (B) 35 (C) 40 (D) 45 (E) 50
-

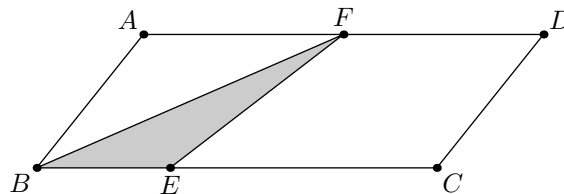
9. You are given a rectangular piece of perspex measuring $50\text{cm} \times 32\text{cm}$, which is then cut into several pieces and rearranged into a square. What is the length of the side of the square?
(A) $\sqrt{182}$ (B) 40 (C) 41 (D) 120 (E) 1600
-

10. John drives from Johannesburg to Cape Town at an average speed of 90 kilometers per hour, and he drives back at an average speed of 110 kilometers per hour. What is John's average speed for the whole journey (in km/h)?
(A) 98 (B) 99 (C) 100 (D) 101 (E) Impossible to determine.
-

11. If a is smaller than b , c is smaller than d and b is smaller than d , then which number is the smallest?
(A) a (B) b (C) c (D) d (E) Impossible to determine.
-

12. $ABCD$ is a parallelogram, F is the midpoint of AD and E is a point on BC such that $BE : EC = 1 : 3$.

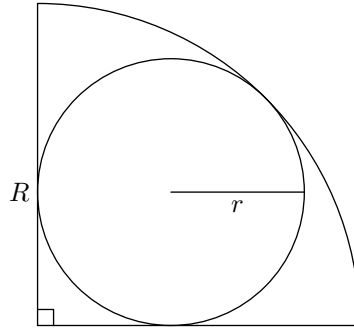
Calculate the value of $\frac{\text{area of } \triangle BEF}{\text{area of parallelogram } ABCD}$.



- (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{3}{7}$ (E) $\frac{1}{2}$
-

13. There are five sticks measuring 1cm, 2cm, 3cm, 4cm and 5cm. How many different triangles can one form using three sticks at a time?
(A) 2 (B) 3 (C) 5 (D) 7 (E) 9
-

14. The radius r of the small circle is 1cm. Determine the radius R of the large (quarter) circle.



- (A) $\frac{3\sqrt{2}}{2}$ (B) $1 + \sqrt{2}$ (C) $\frac{5}{2}$ (D) $2\sqrt{2}$ (E) 3
-

15. The set of odd numbers are arranged as follows:

$$\begin{array}{cccc}
 & & 1 & \\
 & 3 & 5 & 7 \\
 9 & 11 & 13 & 15 & 17 \\
 \vdots & \vdots & \vdots & &
 \end{array}$$

What is the middle number of the 20th row?

- (A) 759 (B) 761 (C) 763 (D) 765 (E) 767
-

Part C: Six marks each.

16. In the sequence of numbers 1, 2, 3, ..., 2008, it is possible to choose two (different) numbers whose sum is divisible by 11. A new sequence is formed by excluding some numbers from the original sequence in such a way that it is impossible to choose two numbers from the new sequence whose sum is divisible by 11. What is the maximum number of numbers in this sequence?

(A) 910 (B) 911 (C) 915 (D) 916 (E) 1097

17. Two intersecting straight lines divide the 2-dimensional plane into 4 parts, and three straight lines (intersecting in different points) divide the plane into 7 parts. How many lines will divide the plane into 172 parts? (Assume that no two lines are parallel and that no three lines pass through the same point.)

(A) 11 (B) 16 (C) 17 (D) 18 (E) 19

18. In a certain suburb, the power supply is interrupted during peak hours on average once every 7 days, and the power supply is interrupted during off-peak hours on average once every 17 days. On some days, the power supply is interrupted both during peak hours and off-peak hours. Peak hours are between 6am and 9am and again between 5pm and 9pm. What is the probability that the suburb's power supply is interrupted on a given day?

(A) $\frac{1}{119}$ (B) $\frac{11}{288}$ (C) $\frac{1}{12}$ (D) $\frac{23}{119}$ (E) $\frac{24}{119}$

19. Jeremy can build a wall in 16 hours if he works alone. Mpume can build the same wall in 12 hours if she works alone. If they work together they can build the wall in 8 hours, but because they sometimes get in each other's way, they build 16 bricks less per hour than they would if they did not get in each other's way. How many bricks are there in the wall?

(A) 867 (B) 687 (C) 876 (D) 678 (E) 768

20. A rectangular box with integral dimensions (i.e. its side-lengths are integers) has a volume of 288 cubic units and a surface area of 288 square units. What is the sum of the side-lengths of the box (in units)?

(A) 16 (B) 21 (C) 22 (D) 23 (E) 31

Formula and information sheet

- (a) The natural numbers are 1, 2, 3, 4, 5, ...
(b) The whole numbers (counting numbers) are 0, 1, 2, 3, 4, 5, ...
(c) The integers are ..., -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...
-

- In the fraction $\frac{a}{b}$, a is called the numerator and b the denominator.
-

- (a) Exponential notation:

$$2 \times 2 \times 2 \times 2 \times 2 = 2^5$$

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^6$$

$$a \times a \times a \times a \times \cdots \times a = a^n \quad (n \text{ factors of } a)$$

(a is the base and n is the index (exponent))

- (b) Factorial notation:

$$1 \times 2 \times 3 \times 4 = 4!$$

$$1 \times 2 \times 3 \times \cdots \times n = n!$$

- The area of a

(a) triangle is: $\frac{1}{2} \times (\text{base} \times \text{height}) = \frac{1}{2}(b \cdot h)$;

(b) rectangle is: length \times breadth = lb ;

(c) square is: side \times side = s^2 ;

(d) rhombus is: $\frac{1}{2} \times (\text{product of diagonals})$;

(e) trapezium is: $\frac{1}{2} \times (\text{sum of parallel sides}) \times \text{height}$;

(f) circle is: πr^2 (r = radius).

- The surface area of a

(a) rectangular prism is: $2lb + 2bh + 2hl$ (h = height);

(b) sphere is: $4\pi r^2$;

(c) right prism is: (perimeter of base $\times h$) + (2 \times area of base).

- The perimeter of a

(a) rectangle is: $2 \times \text{length} + 2 \times \text{breadth} = 2l + 2b$;

(b) square is: $4s$;

(c) circle (its *circumference*) is: $2\pi r$.

7. The volume of a

(a) cube is: $s \times s \times s = s^3$;

(b) rectangular prism is: $l \times b \times h$;

(c) cylinder is: $\pi r^2 h$;

(d) right prism is: (area of base) \times (perpendicular height)
= (area of cross-section) \times (perpendicular height).

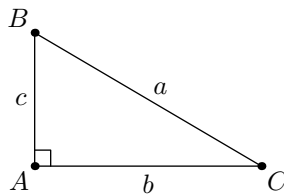
8. The sum of the interior angles of a polygon is equal to $180^\circ \times (n - 2)$,
where n = number of sides.

9. Distance = speed \times time ($d = s \times t$)

Speed = distance \div time ($s = \frac{d}{t}$)

Time = distance \div speed ($t = \frac{d}{s}$)

10. Pythagoras: if $\triangle ABC$ is a right angled triangle, then $a^2 = b^2 + c^2$.



11. Conversions:

$$1 \text{ cm}^3 = 1 \text{ ml}$$

$$1000 \text{ cm}^3 = 1 \ell$$

$$1000 \text{ m} = 1 \text{ km}$$

$$1000 \text{ g} = 1 \text{ kg}$$

$$100 \text{ cm} = 1 \text{ m}$$