SOUTH AFRICAN MATHEMATICS OLYMPIAD
Junior Grade 9 Round 1 2011
Solutions

1. **B** $2 + 3 \times 10 = 2 + 30 = 32$

2. **D** $0.014 \times 0.4 = 0.0056$

3. **E** The fractions are $1-\frac{1}{10}$; $1+\frac{1}{13}$; $1-\frac{1}{20}$; $1+\frac{1}{120}$; $1-\frac{1}{212}$ so the last one differs from 1 by least.

4. **C** $2030 \div 7 = 290$

5. **B** 8 km is 5 miles, and $120 = 8 \times 15$, so 120 km is $5 \times 15 = 75$ miles.

6. **A** The cricket ball rotates three times for each rotation of the soccer ball. So the soccer ball must have three times the circumference of the cricket ball, and therefore three times its radius, i.e. $3 \times 4 = 12$ cm.

7. **B** The eleven numbers have a sum of $11 \times 18 = 198$. When 42 is added we have twelve numbers with a total of 240, so their average must be $240 \div 12 = 20$.

8. **D** The exterior angle of a triangle is the sum of the opposite interior angles, so $y + x + 10° = 2x - 40°$ and thus $y = x - 50°$

9. **E** The LCM of 4 and 6 is 12, so if the competitions are both held in the same year, it will be twelve years before they are again in the same year. From 1968 to 2004 is 3×12, and from 1968 to 2196 is 19×12, so the required number is $19 - 3 + 1 = 17$.

10. **D** By Pythagoras, $\text{width}^2 = 65^2 - 63^2$, which is $(65 + 63)(65 - 63) = 128 \times 2 = 256 = 16^2$

11. **C** Pairing the terms shows the sum is $(1 - 2) + (3 - 4) + \ldots + (2009 - 2010) + 2011$, which is $-1 - 1 - 1 \ldots + 2011$, with the -1 appearing $2010/2 = 1005$ times. So the sum is $2011 - 1005 = 1006$.
Alternatively, pair the numbers thus: $1 + (-2 + 3) + (-4 + 5) + \ldots + (-2010 + 2011)$, obtaining $1 + 1 + \ldots$ with $1 + \frac{1}{2}(2010)$ 1s, totalling 1006.

12. **E** The last digits of the numbers given are 1–1, 9–4 etc. All end in 0 or 5 except for the last one which ends in 3.

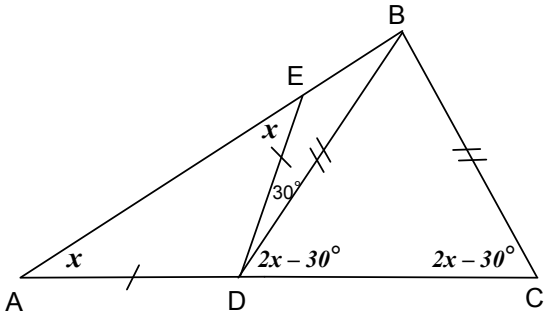
13. **E** Continuing the pattern a bit further shows clearly that the last digits in the columns simply alternate, with all the numbers ending in 3 in the last column

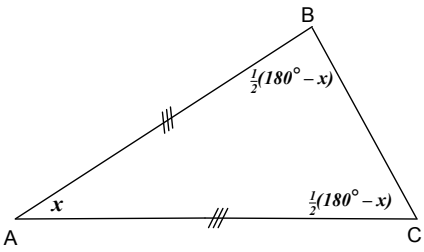
A	**B**	**C**	**D**	**E**
1	4	7	10	13
28	25	22	19	16
31	34	37	40	43
		…	49	46

14. **C** x increases to $1.50x$ and the $2x$ becomes $0.70 \times 2x = 1.40x$. So the difference is $1.50x - 1.40x = 0.1\,x$

15. **D** The numbers starting with 3 are 30, 31, 32, …, 39 (ten of these), and the numbers ending with 3 are 3, 13, 23, …, 93 (ten of these). However, 33 has been counted twice but should not be counted at all. So there are 20 – 2 = 18 numbers that satisfy the condition, and the probability is thus 18 ÷ 99 = 2/11.

16. **D**





If $B\hat{A}C = x$ then $A\hat{E}D = x$ (Δ ADE is isosceles)
Then $B\hat{D}C = 2x - 30°$ (ext. angle of triangle)
so $B\hat{C}A = 2x - 30°$ (Δ BDC is isosceles).
Now $A\hat{B}C$ = 2*x* – 30° (Δ ABC is isosceles),
so 5*x* – 60° = 180° (angle sum of Δ ABC), giving 5*x* = 240° and so *x* = 48°

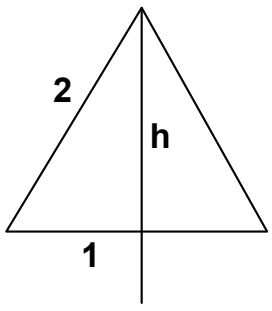
Or as alternative calculate angle BCA as above, but then we can directly from the two sketches see that 2x – 30° = ½(180°- *x*), which similarly solves to *x* = 48°.

17. **E** If X is the boy's age now, the man's age now is 2X. Fifteen years ago their ages were X – 15 and 2X – 15, so 2X – 15 = 3(X – 15). This gives 2X – 15 = 3X – 45, and hence X = 30.

18. **A** We start with 45 litres wine, 0 litres water. Then that becomes 36 litres wine, 9 litres water (ratio 4:1). The removed ten litres of mixture will therefore contain 8 litres wine, 2 litres water, so when ten litres of water is added the tank finally contains 28 litres wine, 17 litres water.

19. **C** The three sectors have the same area as half a circle, and since the radius is 1 (half a side of the triangle) that area is π/2.

The triangle has base 2 and so height $\sqrt{3}$ (by Pythagoras), which means its area is $1 \times \sqrt{3} = \sqrt{3}$, and the shaded area is found by subtracting the area of the sectors from that of the triangle.



20. **C** The pentagons have a total of 12 × 5 = 60 edges, and the hexagons have a total of 20 × 6 = 120 edges. All these 180 edges must be joined in pairs at the seams, so 90 pairings, i.e. seams, are needed.