THE HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

FIRST ROUND 2006
JUNIOR SECTION: GRADES 8 AND 9

SOLUTIONS AND MODEL ANSWERS

NUMBER	POSITION
1	B
2	D
3	E
4	E
5	B
6	A
7	C
8	C
9	D
10	B
11	A
12	C
13	E
14	D
15	A
16	C
17	E
18	B
19	A
20	D

1. **ANSWER**: B
SOLUTION:

$$\begin{aligned}&6\times111-3\times111\\&=666-333\\&=333\end{aligned}$$

or

$$\begin{aligned}&6\times111-3\times111\\&=3\times111\\&=333\end{aligned}$$

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2. **ANSWER**: D
SOLUTION:

$\frac{1}{3}$; 31%; $\frac{3}{10}$; 0,313; 0, 303

0,333…; 0,310; 0,300; 0,313; 0,303

Rearranging from lowest to highest: 0,300; 0,303; 0,310; 0,313; 0,333…

i.e. $\frac{3}{10}$; 0,303; 31%; 0,313; $\frac{1}{3}$

Middle number = 31%

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3. **ANSWER**: E
SOLUTION:

4 points earned from R75

$\therefore$ 36 points earned from $\text{R}\frac{75}{1}\times\frac{36}{4}=\text{R}675$

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4. **ANSWER**: E
SOLUTION:

$$\begin{aligned}&\left(\frac{1}{2}\times\frac{1}{2}\right)\div\frac{1}{3}\\&=\frac{1}{4}\times\frac{3}{1}\\&=\frac{3}{4}\end{aligned}$$

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5. **<u>ANSWER</u>: B**
<u>SOLUTION</u>:

The lowest common multiple of 8, 12 and 30:

$8 = 2^3$
$12 = 2^2 \times 3$
$30 = 2 \times 3 \times 5$

l.c.m. of $(8,\ 12,\ 30) = 2^3 \times 3 \times 5$
$= 8 \times 15$
$= 120$

OR

2	8; 12; 30
2	4; 6; 15
2	2; 3; 15
3	1; 3; 15
5	1; 1; 5
	1; 1; 1

l.c.m. $= 2 \times 2 \times 2 \times 3 \times 5$
$= 120$

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6. **<u>ANSWER</u>: A**
<u>SOLUTION</u>:

7777×9999
$= 7777 \times (10\ 000 - 1)$
$= 77\ 770\ 000 - 7777$
$= 77\ 762\ \ 223$
The hundred's digit is: 2

OR

$7 \times 9 = 63\ \ (2)$
$77 \times 99 = 77 \times (100 - 1) = 7623\ \ (4)$
$777 \times 999 = 777 \times (1000 - 1) = 776\ 223\ \ (6)$

Therefore: 7777×9999
$= 77\ 762\ 223$

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7. **<u>ANSWER:</u> C**
<u>SOLUTION</u>:

Robertson family

1	2	3	4		5	6
○	○	○	○		○	○
	sisters				**brothers**	

If any sister talks she will say, “I have 3 sisters and 2 brothers.”
If a brother talks, he will say, “I have 1 brother and 4 sisters.”

The **<u>minimum</u>** number of children in the family: 6

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8. **ANSWER: C**
SOLUTION:

$$\left(4^2 = 16\right), 17, 18, \cdots, 62, 63, \left(64 = 4^3\right)$$

The number of whole numbers between 4^2 and 4^3 is: $64 - 16 - 1 = 47$

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9. **ANSWER: D**
SOLUTION:

Since the hour hand is $\frac{2}{5}$ of the distance between 4 and 5, the number of minutes past 4 o'clock is

$$\frac{2}{5} \times 60 = 24$$

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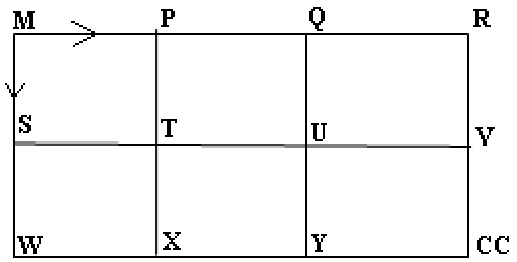
10. **ANSWER: B**
SOLUTION:

The possible routes are:

MPQRVCC, MPQUVCC, MPQUYCC, MPTUVCC, MPTUYCC, MPTXYCC, MSTUVCC, MSTUYCC, MSTXYCC, MSWXYCC

There are 10 possible routes:



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11. **ANSWER: A**
SOLUTION:

$$\begin{aligned} & 12^2 \times (4 \times 3) \\ &= 12 \times 12 \times 12 \\ &= 12^3 \\ &\therefore n = 12 \end{aligned}$$

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12. **ANSWER: C**
SOLUTION:

At worst:
Suppose the lady draws 6 beads, she could have
6 beads = 2 green + 2 white + 2 black.
Suppose she draws 1 more bead.
This bead will not be a green. Why?
This bead will definitely be a white or a black.
In this case we will obtain 3 of the same colour.

Answer: 7 beads.

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13. **ANSWER: E**
SOLUTION:

During equal time intervals, equal distances are covered.
$\therefore$ The answer is (E): travelling at a constant speed, where

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

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14. **ANSWER: D**
SOLUTION:

Let x and $x+1$ be two consecutive numbers.
Then $x(x+1) = p$

$$\therefore x^2 + x = p \quad \ldots \quad (1)$$

$$\begin{aligned} \text{and} \quad (x+1)^2 - x &= (x+1)(x+1) - x \\ &= x^2 + x + x + 1 - x \\ &= x^2 + x + 1 \\ &= p + 1 \end{aligned}$$

OR

Using a pattern of two consecutive numbers:

e.g. $\begin{cases} 1 \times 2 = (2) \\ 2^2 - 1 = 3 = 2 + 1 \end{cases}$ $\begin{cases} 2 \times 3 = (6) \\ 3^2 - 2 = 7 = 6 + 1 \end{cases}$ $\begin{cases} 3 \times 4 = (12) \\ 4^2 - 3 = 13 = 12 + 1 \end{cases}$

If 2, 6, 12 etc. are p then the answer $= p + 1$

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15. **ANSWER: A**
SOLUTION:

The weights she can use are:
1 kg, 2 kg, 4 kg and 8 kg
Combinations are:

$$T_1 = 1 \text{ kg}$$
$$T_2 = 2 \text{ kg}$$
$$T_3 = 1 + 2 = 3 \text{ kg}$$
$$T_4 = 4 \text{ kg}$$
$$T_5 = 1 + 4 = 5 \text{ kg}$$
$$T_6 = 2 + 4 = 6 \text{ kg}$$
$$T_7 = 1 + 2 + 4 = 7 \text{ kg}$$
$$T_8 = 8 \text{ kg}$$
$$T_9 = 1 + 8 = 9 \text{ kg}$$
$$T_{10} = 2 + 8 = 10 \text{ kg}$$
$$T_{11} = 1 + 2 + 8 = 11 \text{ kg}$$
$$T_{12} = 4 + 8 = 12 \text{ kg}$$
$$T_{13} = 1 + 4 + 8 = 13 \text{ kg}$$
$$T_{14} = 2 + 4 + 8 = 14 \text{ kg}$$
$$T_{15} = 1 + 2 + 4 + 8 = 15 \text{ kg}$$

$\therefore$ 15 weight combinations.

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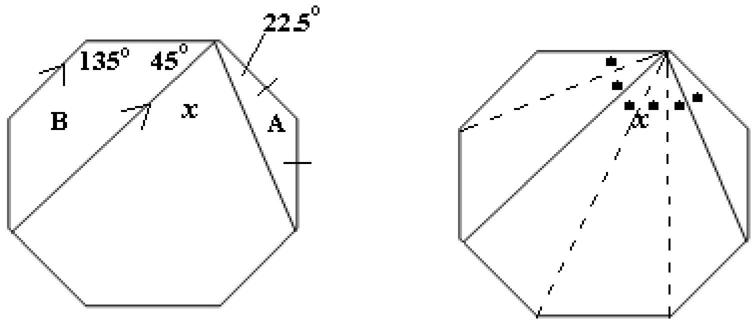
16. **ANSWER: C**
SOLUTION:



Each angle of octagon $= 135^\circ$.
Triangle A: base angle $= 22{,}5^\circ$
Triangle B: angle $= 45^\circ$ parallel lines.

$$x = 135^\circ - 45^\circ - 22{,}5^\circ = 67{,}5^\circ$$

OR

Each angle of octagon is 135°.

$$x = 3 \times \frac{1}{6} \times 135^\circ = 67{,}5^\circ.$$

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17. **ANSWER: E**
SOLUTION:

$$\frac{1}{1\times2}=\frac{1}{2}$$

$$\frac{1}{1\times2}-\frac{1}{2\times3}=\frac{1}{2}-\frac{1}{6}=\frac{3}{6}-\frac{1}{6}=\frac{2}{6}=\frac{1}{3}$$

$$\frac{1}{1\times2}-\frac{1}{2\times3}-\frac{1}{3\times4}=\frac{1}{3}-\frac{1}{12}=\frac{4}{12}-\frac{1}{12}=\frac{3}{12}=\frac{1}{4}$$

$$\frac{1}{1\times2}-\frac{1}{2\times3}-\frac{1}{3\times4}-\cdots-\frac{1}{49\times50}=\frac{1}{50}$$

$$\therefore \text{ Answer: } \frac{1}{50}$$

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18. **ANSWER: B**
SOLUTION:

		Number in each row
R1:	1	1
R2:	2 3	2
R3:	4 5 6	3
R4:	7 8 9 10	4
R5:	11 12 13 14 15	5

Last no. $R1=\frac{1\times2}{2}=1$

$$R2=\frac{2\times3}{2}=3$$

$$R3=\frac{3\times4}{2}=6$$

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$$R50=\frac{50\times51}{2}=1275$$

Middle no. of 51st row = 1275 + 26
= 1301 (answer)

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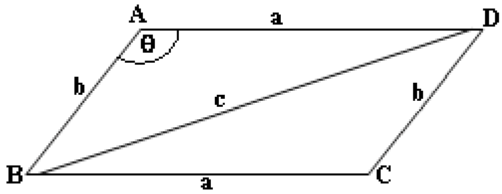
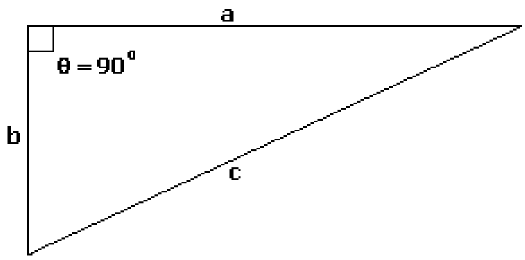
19. **<u>Answer</u>: A**

<u>Solution</u>:



$$2a + 2b + c = 42 \qquad \therefore\ c = 2(21 - a - b)$$

So c must be even.



If $\theta = 90^\circ$ then $c^2 - (a^2 + b^2) = 0$

But we want c bigger than this

$\therefore$ check that $\boxed{c^2 > a^2 + b^2}$ and $\boxed{c < a + b}$.

If $c = 2, 4,$ or 6 then $c^2 > a^2 + b^2$ is not satisfied.

If $c = 8,$ then $a + b = 17.$

$$5 + 12 = 6 + 11 = 7 + 10 = 8 + 9 = 17 \quad \text{don't work, because } a^2 + b^2 > 8^2.$$

If $c = 10,$ then $a + b = 16$

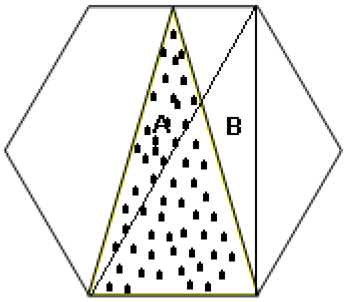
$$8 + 8 = 7 + 9 = 6 + 10 = 5 + 11, \text{ etc.} \quad \text{don't work.}$$

If $c = 12,$ then $a + b = 15$

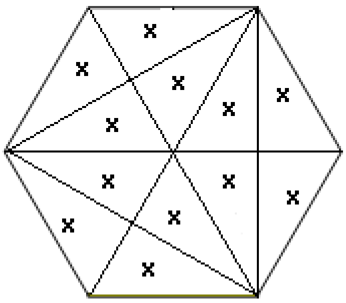
$$\underbrace{8 + 7 = 9 + 6 = 10 + 5 = 11 + 4}_{\text{work}} = 12 + 3 = 13 + 2$$

If $c = 14,$ then $a + b = 14,$ won't work.

20. **<u>Answer</u>: D**
<u>Solution</u>:



Area of A = area of B
(heights equal; base common)
Now complete as in figure (2).
Use lines of symmetry
Represent each area as '*x*'



Fraction shaded $= \frac{4x}{12x}$

$= \frac{1}{3}$