HARMONY SOUTH AFRICAN MATHEMATICS OLYMPIAD

Organised by the SOUTH AFRICAN MATHEMATICS FOUNDATION.
Sponsored by HARMONY GOLD MINING.

FIRST ROUND 2008
JUNIOR SECTION: GRADES 8 AND 9
18 MARCH 2008
TIME: 60 MINUTES
NUMBER OF QUESTIONS: 20

Instructions:

1. Do not open this booklet until told to do so by the invigilator.
2. This is a multiple choice question paper. Each question is followed by answers marked A, B, C, D and E. Only one of these is correct.
3. Each correct answer is worth 5 marks. There is no penalty for an incorrect or an unanswered question.
4. You must use an HB pencil.
   Rough paper, a ruler and an eraser are permitted.
   Calculators and geometry instruments are not permitted.
5. Diagrams are not necessarily drawn to scale.
6. The centre page is an information and formula sheet. Please tear it out for your use.
7. Indicate your answers on the sheet provided.
8. Start when the invigilator tells you to do so.
   You have 60 minutes to complete the question paper.
9. Answers and solutions will be available at www.samf.ac.za

DO NOT TURN THE PAGE
UNTIL YOU ARE TOLD TO DO SO.
DRAAI DIE BOEKIE OM VIR DIE AFRIKAANSE VRAESTEL

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Organisations involved: AMESA, SA Mathematical Society, SA Akademie vir Wetenskap en Kuns

1. The value of $0,4 \div 4$ is…

   A. 0,1 B. 0,2 C. 0,4 D. 0,8 E. 1,2

2. Give an estimate of the following:

$$\frac{2008 \times 1710}{3421}$$

   A. 1080 B. 1060 C. 1040 D. 1020 E. 1000

3. Find the value of 3% x R2008 + 7% x R2008.

   A. R2008,00 B. 200,80 C. 20,08 D.280,00 E. R200,08

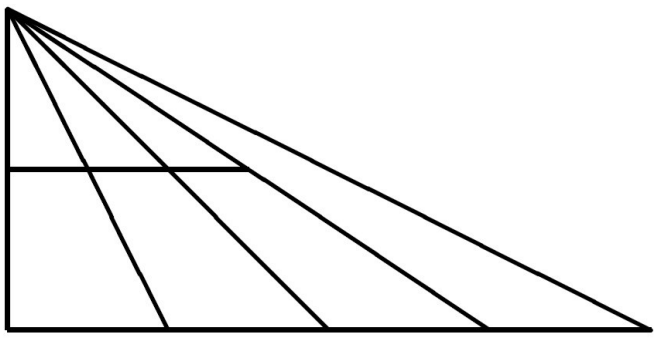
4. If 2918 x 13 278 = 38 745 204
   and 2918 x 13 129 = 38 310 422,
   then find the value of 2 918 x 149.

   A. 434 782 B. 434 792 C. 434 882 D. 434 786 E. 434 888

5. How many numbers between 100 and 500 are divisible by both 6 and 9 ?

   A. 19 B. 20 C. 21 D. 22 E. 23

6. How many triangles of different sizes are there in the following figure ?



   A. 7 B. 16 C. 20 D. 22 E. 24

7. Five protractors cost the same as seven set squares. The cost of five protractors and a set square is equal to R62,00.
   What would one pay for 28 such set squares?

   A. R197 B. R207 C. R217 D. R227 E. R237

8. A group of girls share 150 apples equally and 105 doughnuts equally. The largest number of girls in this group is...

A. 6 B. 9 C. 12 D. 15 E. 18

9. How many natural numbers n are there such that n^2 lies between 101 and 300 ?

A. 3 B. 4 C. 5 D. 6 E. 7

10. Consider the following numbers:-

16; 13; 10; 8.

Two numbers are selected from the above set and added. The remaining two numbers are selected from the above set and added. These possible sums are subtracted. How many different positive answers are possible?

A. 7 B. 6 C. 5 D. 4 E. 3

11. Consider the following 4 numbers:

$a; b; c; d$

b is greater than a by $\frac{1}{5}$,

b is less than c by $\frac{1}{6}$ and

d is greater than a by $\frac{1}{2}$.

If $a + b + c + d = 9\frac{1}{15}$ then find the value of a.

A. 1 B. 2 C. 3 D. 4 E. 5

12. Tickey and Sixpense collected money for charity in the ratio of 5:2 respectively. If Sixpense collected R600 less than Tickey then what was the total amount collected by Tickey and Sixpense ?

A. R1050 B. R1120 C. R1400 D. R1750 E. R1820

13. The digit 5 is written between the digits of a two-digit number to form a three-digit number. This number is 410 more than the original two-digit number. If the sum of the digits of the three-digit number is 12, then what is the difference between the digits of the two-digit number ?

A. 0 B. 1 C. 3 D. 5 E. 7

14. Find the value of $399 - 397 + 395 - 393 + \ldots\ldots + 83 - 81$.

A. 154 B. 156 C. 158 D. 160 E. 162

15. What is the smallest positive integer n such that the following product ends in six zeros?

16 x 34 x 75 x 21 x 13 x n

A. 125 B. 625 C. 1250 D. 10 000 E. 1 000 000

16. Determine the value of

$$\sqrt{(1-\frac{1}{5})(1-\frac{1}{6})(1-\frac{1}{7})\ldots(1-\frac{1}{400})}\,.$$

A. $\frac{1}{50}$ B. $\frac{1}{40}$ C. $\frac{1}{20}$ D. $\frac{1}{10}$ E. $\frac{1}{5}$

17. The dimensions of a rectangular floor are 710cm by 430cm. What is the maximum number of whole tiles with dimensions 25cm by 20cm which will fit onto this floor without changing the direction of the tiles ?

A. 588 B. 590 C. 595 D. 596 E. 598

18. Consider:

1
3 5 7
9 11 13 15 17
19 21 23 25 27 29 31
……..

What is the middle number of the 61st row?

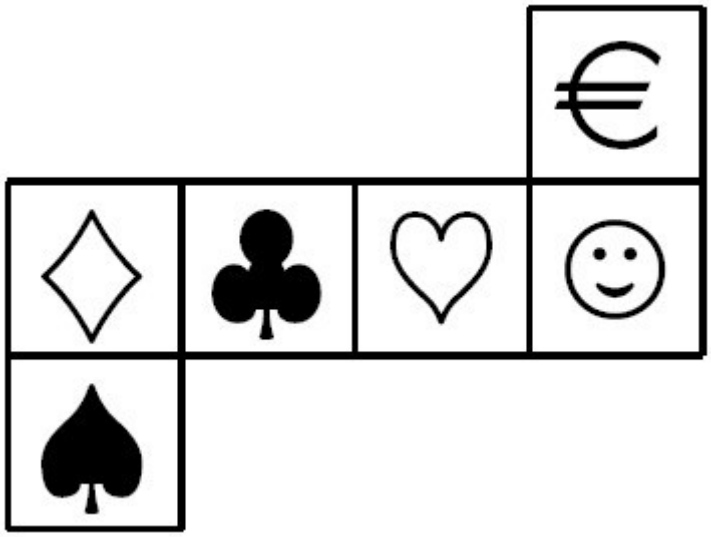
A. 7121 B. 7221 C. 7321 D. 7421 E. 7521

19. #[65^2]= #[4 225] = 4+2+2+5=13 and #[665^2]= 19, where # [x^2] gives the sum of the digits of x^2 (x consists of a certain number of 6's followed by a 5).

Find x if #[x^2]= 49.

A. 66 665 B. 666 665
C. 6 666 665 D. 66 666 665
E. 666 666 665

20. The net of a cube is given:



Some of the cubes below can be obtained by folding the above net (ignoring the orientation of the pictures).

(1) (2) (3) (4) (5)

Which one of the following statements is true?

A. Only cube (3) can be obtained.
B. Only cube (4) cannot be obtained.
C. Cubes (1), (3) and (4) can be obtained.
D. Cubes (1), (2), (4) cannot be obtained.
E. Cubes (2), (4) and (5) cannot be obtained.

Formula and Information Sheet

1.1 The natural numbers are 1; 2; 3; 4; 5; …

1.2 The whole numbers (counting numbers) are 0; 1; 2; 3; 4; 5; …

1.3 The integers are …; –4; –3; –2; –1; 0; 1; 2; 3; 4; 5; …

2. In the fraction $\frac{a}{b}$, a is called the numerator and b the denominator.

3.1 Exponential notation:

$2\times2\times2\times2\times2=2^5$

$3\times3\times3\times3\times3\times3=3^6$

$a\times a\times a\times a\times \ldots \times a=a^n$ (n factors of a)

(a is the base and n is the index (exponent))

3.2 Factorial notation:

$1\times2\times3\times4=4!$

$1\times2\times3\times \ldots \times n=n!$

4. Area of a

4.1 triangle is: $\frac{1}{2}\times$ (base $\times$ height) $=\frac{1}{2}(b.h)$

4.2 rectangle is: length $\times$ width = lw

length $\times$ breadth = lb

4.3 square is: side $\times$ side = s^2

4.4 rhombus is: $\frac{1}{2}\times$ (product of diagonals)

4.5 trapezium is: $\frac{1}{2}\times$(sum of parallel sides) $\times$ height

4.6 circle is: πr^2 (r = radius)

5. Surface area of a:

5.1 rectangular prism is: $2lb+2lh+2bh$ (h = height)

5.2 sphere is: $4\pi r^2$

6. Perimeter of a:

6.1 rectangle is: $2\times$length $+2\times$breadth

$2l+2b$

or $2l+2w$ (w = width)

6.2 square is: $4s$

7. Circumference of a circle is: $2\pi r$

8. Volume of a:

8.1 cube is: $s\times s\times s=s^3$

8.2 rectangular prism is: $l \times b \times h$

8.3 cylinder is: $\pi r^2 h$

9.1 Volume of a right prism is: area of cross-section × perpendicular height

or area of base × perpendicular height

9.2 Surface area of a right prism is: (perimeter of base × h) + (2 × area of base)

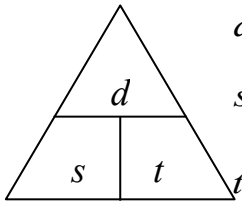
10. Sum of the interior angles of a polygon is: $180^\circ(n-2)$ [n = number of sides]

11.

Distance = speed × time ($d = s \times t$)

Speed = distance ÷ time ($s = \frac{d}{t}$)

Time = distance ÷ speed ($t = \frac{d}{s}$)

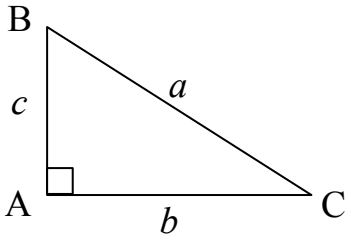


$d = s \times t$

$s = \frac{d}{t}$

$t = \frac{d}{s}$

12. Pythagoras:



If ΔABC is a right-angled triangle, then $a^2 = b^2 + c^2$.

13. Conversions:

1 cm^3 = 1 mℓ ; 1000 cm^3 = 1 ℓ

1000 m = 1 km ; 1000 g = 1 kg ; 100 cm = 1 m